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Lesson 4: Properties of Exponents and Radicals

Student Outcomes

* Students rewrite expressions involving radicals and rational exponents using the properties of exponents.

Lesson Notes

In Lesson 1, students reviewed the properties of exponents for integer exponents before establishing the meaning of the th root of a positive real number and how it can be expressed as a rational exponent in Lesson 3. In Lesson 4, students extend properties of exponents that applied to expressions with integer exponents to expressions with rational exponents. In each case, the notation specifically indicates the principal root (e.g., is , as opposed to ).

This lesson extends students’ thinking using the properties of radicals and the definitions from Lesson 3 so that they can see why it makes sense that the properties of exponents hold for any rational exponents (**N-RN.A.1**). Examples and exercises work to establish fluency with the properties of exponents when the exponents are rational numbers and emphasize rewriting expressions and evaluating expressions using the properties of exponents and radicals (**N-RN.A.2**).

Classwork

Opening (2 minutes)

*Scaffolding:*

* Throughout the lesson, remind students of past properties of integer exponents and radicals either through an anchor chart posted on the wall or by writing relevant properties as they come up. Included is a short list of previous properties used in this module.
* For all real numbers *,* , and all integers ,:

Students revisit the properties of square roots and cube roots studied in Module 1 to remind them that we extended those to any th root in Lesson 3. So, they are now ready to verify that the properties of exponents hold for rational exponents.

Draw students’ attention to a chart posted prominently on the wall or to their notebooks where the properties of exponents and radicals are displayed, including those developed in Lesson 3.

Remind students of the description of exponential expressions of the form , which they will be making use of throughout the lesson:

Let be any positive real number, and , be any integers with ; then and .

Opening Exercise (5 minutes)

These exercises briefly review content from Module 1 and the last lesson.

Opening Exercise

Write each exponent as a radical, and then use the definition and properties of radicals to write that expression as an integer.

To transition from the Opening Exercise to Example 1, ask students to write the first two problems above in exponent form. Then, ask them to discuss with a partner whether or not it would be true in general that for positive real numbers where , , , and are integers with and .

* How could you write the with rational exponents? How about ?

**MP.3**

**&**

**MP.8**

* + *and*
* Based on these examples, is the exponent property valid when and are rational numbers? Explain how you know.
  + *Since the exponents on the left side of each statement add up to the exponents on the right side, it appears to be true. However, the right side exponent was always If we work with and write it in exponent form, we would get (after noting ) that . So, it appears to be true in general. Note that examples alone do not prove that a mathematical statement is always true.*

In the rest of this lesson, we will make sense of these observations in general to extend the properties of exponents to rational numbers by applying the definition of the root *of* and the properties of radicals introduced in Lesson 3.

Examples 1–3 (10 minutes)

In the previous lesson, we assumed that the exponent property for positive real numbers and integers and would also hold for rational exponents when the exponents were of the form , where was a positive integer. This example will help students see that the property below makes sense for any rational exponent:

**MP.7**

**&**

**MP.8**

, where , , , and are integers with and .

Perhaps model Example 1 below and then have students work with a partner on Example 2. Make sure students include a justification for each step in the problem. When you get to Example 3, be sure to use the following discussion questions to guide students.

* How can we write these expressions using radicals?
  + *In Lesson* 3*, we learned that and for positive real numbers and positive integers and .*
* Which properties help us to write the expression as a single radical?
  + *The property of radicals that states for positive real numbers and and positive integer , and the property of exponents that states for positive real numbers and integers and .*
* How do we rewrite this expression in exponent form?
  + *In Lesson* 3*, we related radicals and rational exponents by .*
* What makes Example 3 different from Examples 1 and 2?
  + *The exponents have different denominators, so when we write the expression in radical form, the roots are not the same, and we cannot apply the property that .*
* Can you think of a way to rewrite the problem so it looks like the first two problems?
  + *We can write the exponents as equivalent fractions with the same denominator.*

Examples 1–3

*Scaffolding:*

* Throughout the lesson, you can create parallel problems to demonstrate that these problems work with numerical values as well.
* For example, in part (a) substitute for .
* In part (b), substitute a perfect cube such as or for

Write each expression in the form for positive real numbers and integers and with by applying the properties of radicals and the definition of th root.

By the definition of th root,

The rational number is equal to . Thus,

Thus,

Write the exponents as equivalent fractions with the same denominator.

Rewrite in radical form.

Rewrite as a single radical expression.

Rewrite in exponent form using the definition.

Thus,

* Now add the exponents in each example. What is ? ? ?
  + *, , and .*
* What do you notice about these sums and the value of the exponent when we rewrote each expression?
  + *The sum of the exponents was equal to the exponent of the answer.*

Based on these examples, particularly the last one, it seems reasonable to extend the properties of exponents to hold when the exponents are any rational number. Thus, we can state the following property.

* For any integers , , and, with and , and any real numbers so that and are defined,

Have students copy this property into their notes along with the ones listed below. You can also write these properties on a piece of chart paper and display them in your classroom. These properties are listed in the lesson summary.

In a similar fashion, the other properties of exponents can be extended to hold for any rational exponents as well.

* For any integers , , and, with and , and any real numbers and so that , , and are defined,

At this point, you might have your class look at the opening exercise again and ask them which property could be used to simplify each problem.

For advanced learners, a derivation of the property we explored in Example 1 is provided below.

Rewrite and as equivalent exponential expressions in which the exponents have the same denominator, and apply the definition of the as the th root.

By the definition of and then using properties of algebra, we can rewrite the exponent to be .

Exercises 1–4 (6 minutes)

*Scaffolding:*

* When students get to these exercises, you may need to remind them that it is often easier to rewrite aswhen you are trying to evaluate radical expressions.
* For students who struggle with arithmetic, you can provide a scientific calculator, but be sure to encourage them to show the steps using the properties.

Have students work with a partner or in small groups to complete these exercises. Students are rewriting expressions with rational exponents using the properties presented above. As students work, emphasize that we do not need to write these expressions using radicals since we have just established that we believe that the properties of exponents hold for rational numbers. In the last two exercises, students will have to use their knowledge of radicals to rewrite the answers without exponents.

Exercises 1–4

Write each expression in the form . If a numeric expression is a rational number, then write your answer without exponents.

Example 4 (5 minutes)

* We can rewrite radical expressions using properties of exponents. There are other methods for rewriting radical expressions, but this example models using the properties of exponents. Often, textbooks and exams give directions to simplify an expression, which is vague unless we specify what it means. We want students to develop fluency in applying the properties, so the directions here say to rewrite in a specific fashion.

Example 4

Rewrite the radical expression so that no perfect square factors remain inside the radical.

* Although this process may seem drawn out, once it has been practiced, most of the steps can be internalized and expressions are quickly rewritten using this technique.

Exercise 5 (5 minutes)

Students should work individually or in pairs on this exercise.

Exercise 5

1. If , , and , the following expressions are difficult to evaluate without using properties of radicals or exponents (or a calculator). Use the definition of rational exponents and properties of exponents to rewrite each expression in a form where it can be easily evaluated, and then use that exponential expression to find the value.

Evaluating, we get .

Evaluating, we get .

Exercise 6 (5 minutes)

This exercise will remind students that rational numbers can be represented in decimal form and will give them a chance to work on their numeracy skills. Students should work on this exercise with a partner or in their groups to encourage dialogue and debate. Have a few students demonstrate their results to the entire class. There is more than one possible approach, so when you debrief, try to share different approaches that show varied reasoning. Conclude with one or two strong arguments. Students can confirm their reasoning using a calculator.

Exercise 6

**MP.3**

1. Order these numbers from smallest to largest. Explain your reasoning.

is between and . We can rewrite , which is .

is between and . We can rewrite , which is .

is larger than .

Thus, is clearly the smallest number, but we need to determine if is greater than or less than . To do this, we know that This means that , and , which is greater than

Thus, the numbers in order from smallest to largest are , , and .

Closing (2 minutes)

Lesson Summary

The properties of exponents developed in Grade 8 for integer exponents extend to rational exponents.

That is, for any integers , , , and, with and and any real numbers and so that , , and are defined, we have the following properties of exponents:

Have students summarize the definition and properties of rational exponents and any important ideas from the lesson by creating a list of what they have learned so far about the properties of exponents and radicals. Circulate around the classroom to informally assess understanding. Reinforce the properties of exponents listed below.

Exit Ticket (5 minutes)

Name Date

Lesson 4: Properties of Exponents and Radicals

Exit Ticket

1. Find the exact value of without using a calculator.
2. Justify that using the properties of exponents in at least two different ways.

Exit Ticket Sample Solutions

1. Find the exact value of without using a calculator.
2. Justify that using the properties of exponents in at least two different ways.

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Problem Set Sample Solutions

1. Evaluate each expression if and .

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1. Rewrite each expression so that each term is in the form , where is a real number, is a positive real number, and is a rational number.

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1. Show that is not equal to when and .

When and , the two expressions are and . The first expression simplifies to , and the second simplifies to . The two expressions are not equal.

1. Show that is not equal to when and .

When and , the two expressions are and . The first expression is , and the second one is . The two expressions are not equal.

1. From these numbers, select (a) one that is negative, (b) one that is irrational, (c) one that is not a real number, and (d) one that is a perfect square:

The first number, , is irrational, the second number is a perfect square, the third number, , is negative, and the last number, , is not a real number.

1. Show that the expression is equal to .
2. Express each answer as a power of .
   1. Multiply by .
   2. Multiply by .
   3. Square .
   4. Divide by .
   5. Show that .
3. Rewrite each of the following radical expressions as an equivalent exponential expression in which each variable occurs no more than once.
4. Use properties of exponents to find two integers that are upper and lower estimates of the value of .

and , so

1. Use properties of exponents to find two integers that are upper and lower estimates of the value of .

and , so . Thus, .

1. Kepler’s third law of planetary motion relates the average distance, , of a planet from the Sun to the time, , it takes the planet to complete one full orbit around the Sun according to the equation . When the time, , is measured in Earth years, the distance, , is measured in astronomical units (AU). (One AU is equal to the average distance from Earth to the Sun.)
   1. Find an equation for in terms of and an equation for in terms of .
   2. Venus takes about Earth years to orbit the Sun. What is its average distance from the Sun?
   3. Mercury is an average distance of AU from the Sun. About how long is its orbit in Earth years?