Lesson 4: Properties of Exponents and Radicals

Classwork

Opening Exercise

Write each exponent as a radical, and then use the definition and properties of radicals to write that expression as an integer.

**Examples 1–3**

Write each expression in the form for positive real numbers and integers and with by applying the properties of radicals and the definition of th root.



Exercises 1–4

Write each expression in the form . If a numeric expression is a rational number, then write your answer without exponents.

**Example 4**

Rewrite the radical expression so that no perfect square factors remain inside the radical.

Exercise 5

1. If , , and , the following expressions are difficult to evaluate without using properties of radicals or exponents (or a calculator). Use the definition of rational exponents and properties of exponents to rewrite each expression in a form where it can be easily evaluated, and then use that exponential expression to find the value.

Exercise 6

1. Order these numbers from smallest to largest. Explain your reasoning.

Lesson Summary

The properties of exponents developed in Grade 8 for integer exponents extend to rational exponents.

That is, for any integers , , , and, with and and any real numbers and so that , , and are defined, we have the following properties of exponents:

Problem Set

1. Evaluate each expression if and .

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1. Rewrite each expression so that each term is in the form , where is a real number, is a positive real number, and is a rational number.

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1. Show that is not equal to when and .
2. Show that is not equal to when and .
3. From these numbers, select (a) one that is negative, (b) one that is irrational, (c) one that is not a real number, and (d) one that is a perfect square:
4. Show that the expression is equal to
5. Express each answer as a power of .
   1. Multiply by .
   2. Multiply by .
   3. Square .
   4. Divide by .
   5. Show that
6. Rewrite each of the following radical expressions as an equivalent exponential expression in which each variable occurs no more than once.
7. Use properties of exponents to find two integers that are upper and lower estimates of the value of .
8. Use properties of exponents to find two integers that are upper and lower estimates of the value of .
9. Kepler’s third law of planetary motion relates the average distance, , of a planet from the Sun to the time it takes the planet to complete one full orbit around the Sun according to the equation . When the time, , is measured in Earth years, the distance, is measured in astronomical units (AU). (One AU is equal to the average distance from Earth to the Sun.)
   1. Find an equation for in terms of and an equation for in terms of .
   2. Venus takes about Earth years to orbit the Sun. What is its average distance from the Sun?
   3. Mercury is an average distance of AU from the Sun. About how long is its orbit in Earth years?