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Lesson 2: Base 10 and Scientific Notation

Student Outcomes

* Students review place value and scientific notation.
* Students use scientific notation to compute with large numbers.

Lesson Notes

This lesson reviews how to express numbers using scientific notation. Students first learn about scientific notation in Grade 8 where they express numbers using scientific notation (**8.EE.A.3**) and compute with and compare numbers expressed using scientific notation (**8.EE.A.4**). Refer to Grade 8, Module 1, Topic B to review the approach to introducing scientific notation and its use in calculations with very large and very small numbers. This lesson sets the stage for the introduction of base logarithms later in this module by focusing on the fact that every real number can be expressed as product of a number between and and a power of . In the paper folding activity in the last lesson, students worked with some very small and very large numbers. This lesson opens with these numbers to connect these lessons. Students also compute with numbers using scientific notation and discuss how the properties of exponents can simplify these computations (**N-RN.A.2**). In both the lesson and the problem set, students define appropriate quantities for the purpose of descriptive modeling (**N-Q.A.2**). The lesson includes a demonstration that reinforces the point that using scientific notation is convenient when working with very large or very small numbers and helps students gain some sense of the change in magnitude when we compare different powers of . This is an excellent time to watch the -minute classic film “Powers of ” by Charles and Ray Eames, available at <https://www.youtube.com/watch?v=0fKBhvDjuy0>, which clearly illustrates the effect of adding another zero. The definition of scientific notation from Grade 8, Module 1, Lesson 9 is included after Example 1. You may want to allow students to use a calculator for this lesson.

Classwork

Opening (2 minutes)

In the last lesson, one of the examples gave the thickness of a sheet of gold foil as millionths of a meter. The Exit Ticket had students calculate the size of a square sheet of paper that could be folded in half thirteen times, and it was very large. We will use these numbers in the Opening Exercise. Before students begin the Opening Exercise, briefly remind them of these numbers that they saw in the previous lesson, and tell them that this lesson will provide them with a way to conveniently represent very small or very large numbers. If you did not have an opportunity to share one of the news stories in Lesson 1, you could do that at this time as well.

Opening Exercise (5 minutes)

Students should work these exercises independently and then share their responses with a partner. Ask one or two students to explain their calculations to the rest of the class. Be sure to draw out the meaning of millionths of a meter and how that can be expressed as a fraction. Check to make sure students know the place value for large numbers like billions. Students should be able to work the first exercise without a calculator, but on the second exercise, students should definitely use a calculator. If they do not have access to a calculator, give them the number squared and simply have them write the rounded value.

Opening Exercise

In the last lesson, you worked with the thickness of a sheet of gold foil (a very small number) and some very large numbers that gave the size of a piece of paper that actually could be folded in half more than times.

* 1. Convert millionths of a meter to centimeters and express your answer as a decimal number.
  2. The length of a piece of square notebook paper that could be folded in half times was . Use this number to calculate the area of a square piece of paper that could be folded in half times. Round your answer to the nearest million.

Rounded to the nearest million, the number is .

* 1. Sort the following numbers into products and single numeric expressions. Then match the equivalent expressions without using a calculator.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

Products: , , , , and

Single numeric expressions: , , , , and

is equal to .

is equal to .

is equal to .

is equal to .

is equal to .

As you review these solutions, point out that very large and very small numbers require us to include many digits to indicate the place value as shown in Exercises 1 and 2. Also, based on Exercise 3, it appears that you can use integer powers of to express a number as a product.

Example 1 (7 minutes)

*Scaffolding:*

* Have advanced learners write the number as the product of a number and a power of three different ways.
* To challenge advanced learners, have them make a convincing argument regarding the truth of a statement such as the following:

Every decimal number can be expressed as the product of another decimal number and a power of .

Write the following statement on the board and ask students to consider whether or not they believe it is true. Have them discuss their thoughts with a partner, and then ask for volunteers to explain their thinking.

Students should explain that every nonzero decimal number can be expressed as the product of a number between and and a power of .

* Think of an example of a decimal number between and . Write it down.

**MP.3**

* Think of a power of . Write it down.
  + *or*
* What does the word product mean?
  + *It means the result of multiplying two numbers together.*
* Compute the product of the two numbers you wrote down.

First, have students share their answers with a partner. Then, put a few of the examples on the board. Finally, demonstrate how to reverse the process to express the following numbers as the product of a number between and and a power of . At this point, you can explain to students that when we write numbers in this fashion, we say that they are written using scientific notation. This is an especially convenient way to represent extremely large or extremely small numbers that otherwise would have many zero digits as place holders. Make sure to emphasize that this notation is simply rewriting a numerical expression in a different form, which helps us to quickly determine the size of the number. Students may need to be reminded of place value for numbers less than and writing equivalent fractions whose denominators are powers of . The solutions demonstrate how any number can be expressed using what we call scientific notation.

Example 1

Write each number as a product of a decimal number between and and a power of .

Students may recall scientific notation from previous grades. Take time to review the definition of scientific notation provided below.

* Our knowledge of the integer powers of enable us to understand the concept of scientific notation.
* Consider the estimated number of stars in the universe: . This is a -digit *whole number* with the **leading digit** (the leftmost digit) followed by zeros. When it is written in the form , it is said to be expressed in *scientific* *notation*.
* A positive, finite decimal[[1]](#footnote-1) is said to be written in **scientific notation** if it is expressed as a product , where is a finite decimal so that , and is an integer. That is, is a finite decimal with only a single, nonzero digit to the left of the decimal point. The integer is called the **order of magnitude[[2]](#footnote-2)** of the decimal .

A positive, finite decimal is said to be written in scientific notation if it is expressed as a product , where is a finite decimal number so that , and is an integer.

The integer is called the *order of magnitude* of the decimal .

Exercises 1–6 (4 minutes)

Students should work these exercises independently as you monitor their progress. Encourage students to work quickly and begin generalizing a process for quickly writing numbers using scientific notation (such as counting the number of digits between the leading digit and the ones digit). After a few minutes, share the solutions with students so they can check their work.

Exercises 1–6

For Exercises 1–6, write each decimal in scientific notation.

1. ( zeros after the decimal place)
2. ( zeros after the )
3. ( zeros after the )
4. ( zeros after the decimal place)

To help students quickly write these problems using scientific notation, the number of zeros is written above for each problem. Be very careful that students are not using this number as the exponent on the base . Lead a discussion to clarify that difference for all students who make this careless mistake.

Exercises 7–8 (5 minutes)

After students practice writing numbers in scientific notation, you can really drive the point home that scientific notation is useful when working with very large or very small numbers by showing this demonstration: <http://joshworth.com/dev/pixelspace/pixelspace_solarsystem.html>, which illustrates just how far the planets in our solar system are from each other. After the demonstration, write down the distances between Earth and the Sun, between Jupiter and the Sun, and between Pluto and the Sun on the board, and have students work with a partner to answer Exercise 7. Be sure to mention that these distances are averages; he distances between the planets and the Sun are constantly changing as the planets complete their orbits. The average distance from the Sun to Earth is km. The average distance from the Sun to Jupiter is . The average distance between the Sun and Pluto is. In these exercises, students will round the distances to the nearest tenth to minimize all the writing and help them focus more readily on the magnitude of the numbers relative to one another.

Exercises 7–8

1. Approximate the average distances between the Sun and Earth, Jupiter, and Pluto. Express your answers in scientific notation ( where is rounded to the nearest tenth.
   1. Sun to Earth:
   2. Sun to Jupiter:
   3. Sun to Pluto:
   4. Earth to Jupiter:
   5. Jupiter to Pluto:

1. Order the numbers in Exercise 7 from smallest to largest. Explain how writing the numbers in scientific notation helps you to quickly compare and order them.

**MP.7**

The numbers from smallest to largest are , , , , and . The power of helps to quickly sort the numbers by their order of magnitude, and then it is easy to quickly compare the numbers with the same order of magnitude because they are only written as a number between one and ten.

Example 2 (10 minutes): Arithmetic Operations With Numbers Written Using Scientific Notation

Model the solutions to the following example problems. Be sure to emphasize that final answers should be expressed using the scientific notation convention of a number between and and a power of . On part (a), you may need to provide some additional scaffolding if students are struggling to rewrite the numbers using the same order of magnitude. Have students practice writing a number as a product of a power of in three different ways. For example,  
, etc. The lessons of Algebra I, Module 1, Topic B provide some suggestions and fluency exercises if students need additional practice on arithmetic operations with numbers in scientific notation. Be sure that students understand that the properties of exponents allow you to quickly perform the indicated operations.

*Scaffolding:*

* For Example 2, part (a), students will want to add the exponents as they do when multiplying numbers written using scientific notation. Take time to discuss the differences in the three expressions if you notice students making this type of mistake.

Example 2: Arithmetic Operations with Numbers Written Using Scientific Notation

Debrief with the questions designed to help students see that the order of magnitude and the properties of exponents greatly simplify calculations.

* How do the properties of exponents help to simplify these calculations?
* How can you quickly estimate the size of your answer?

Exercises 9–11 (5 minutes)

Exercises 9–11

1. Perform the following calculations without rewriting the numbers in decimal form.
2. Estimate how many times farther Jupiter is from the Sun than Earth is from the Sun. Estimate how many times farther Pluto is from the Sun than Earth is from the Sun.

***Earth is approximately from the Sun, and Jupiter is approximately from the Sun. Therefore, Jupiter is about times as far from the Sun as Earth is from the sun. Pluto is approximately* km *from the Sun. Therefore, Pluto is approximately times as far from the Sun as Earth is, since***

1. Estimate the distance between Earth and Jupiter and between Jupiter and Pluto.

***The distance between Earth and Jupiter is approximately* , *which is equal to . The distance between Jupiter and Pluto is approximately .***

Closing (3 minutes)

Have students discuss the following question with a partner and record the definition of scientific notation in their mathematics notebook. Debrief by asking a few students to share their responses with the entire class.

* List two advantages of writing numbers using scientific notation.
  + *You do not have to write as many zeros when working with very large or very small numbers, and you can quickly multiply and divide numbers using the properties of exponents.*

Exit Ticket (4 minutes)

Name Date

Lesson 2: Base 10 and Scientific Notation

Exit Ticket

1. A sheet of gold foil is millionths of a meter thick. Write the thickness of a gold foil sheet measured in centimeters using scientific notation.
2. Without performing the calculation, estimate which expression is larger. Explain how you know.

Exit Ticket Sample Solutions

1. A sheet of gold foil is millionths of a meter thick. Write the thickness of a gold foil sheet measured in centimeters using scientific notation.

***The thickness is . In scientific notation, the thickness of a gold foil sheet is , which is .***

1. Without performing the calculation, estimate which expression is larger. Explain how you know.

and

The order of magnitude on the first expression is , and the order of magnitude on the second expression is . The product and quotient of the number between and in each expression is a number between and . Therefore, the second expression is larger than the first one.

Problem Set Sample Solutions

1. Write the following numbers used in these statements in scientific notation. (Note: Some of these numbers have been rounded.)
   1. The density of helium is grams per cubic centimeter.
   2. The boiling point of gold is .
   3. The speed of light is miles per second.
   4. One second is hours.
   5. The acceleration due to gravity on the Sun is .
   6. One cubic inch is cubic yards.
   7. Earth’s population in 2012 was people.
   8. Earth’s distance from the Sun is miles.
   9. Earth’s radius is miles.
   10. The diameter of a water molecule is .
2. **Write** the following numbers in decimal form. (Note: Some of these numbers have been rounded.)
   1. A light year is .
   2. Avogadro’s number is .
   3. The universal gravitational constant is .
   4. Earth’s age is years.
   5. Earth’s mass is .
   6. A foot is miles.
   7. The population of China in 2014was people.
   8. The density of oxygen is grams per liter.
   9. The width of a pixel on a smartphone is .
   10. The wavelength of light used in optic fibers is .
3. State the necessary value of that will make each statement true.

Perform the following calculations without rewriting the numbers in decimal form.

1. The wavelength of visible light ranges from nanometers to nanometers, where . Express the wavelength range of visible light in centimeters.

***Convert to centimeters:***

***The wavelength of visible light in centimeters is to .***

1. In 1694, the Dutch scientist Antonie van Leeuwenhoek was one of the first scientists to see a red blood cell in a microscope. He approximated that a red blood cell was “times as small as a grain of sand.” Assume a grain of sand is wide and a red blood cell is approximately micrometers wide. One micrometer is . Support or refute Leeuwenhoek’s claim. Use scientific notation in your calculations.

*Convert millimeters to meters: . A medium size grain of sand measures   
 across. Similarly, a red blood cell is approximately across. If you divide these numbers, you get*

*So, a red blood cell is times as small as a grain of sand. Leeuwenhoek’s claim was off by approximately a factor of .*

1. When the Mars Curiosity Rover entered the atmosphere of Mars on its descent in 2012, it was traveling roughly . On the surface of Mars, its speed averaged . How many times faster was the speed when it entered the atmosphere than its typical speed on the planet’s surface? Use scientific notation in your calculations.

The speed when it entered the atmosphere is greater than its surface speed by an order of magnitude of .

1. Earth’s surface is approximately water. There is no water on the surface of Mars, and its diameter is roughly half of Earth’s diameter. Assume both planets are spherical. The surface area of a sphere is given by the formula where is the radius of the sphere. Which has more land mass, Earth or Mars? Use scientific notation in your calculations.

***The surface area of Earth is* , *and the surface area of Mars is  
 . Thirty percent of Earth’s surface area is approximately* sq. mi*. Earth has more land mass by approximately .***

1. There are approximately trillion () red blood cells in the human body at any one time. A red blood cell is approximately wide. Imagine if you could line up all your red blood cells end to end. How long would the line of cells be? Use scientific notation in your calculations.

***. That means the line of cells would be* long*. One mile is equivalent to* , *so the line of blood cells measures , which is almost halfway to the moon!***

1. Assume each person needs approximately of living space. Now imagine that we are going to build a giant apartment building that will be mile wide and mile long to house all the people in the United States, estimated to be million people in 2012. If each floor of the apartment building is high; how tall will the apartment building be?

***We need*** ***of living space. .***

*Next, divide the total number of square feet by the number of square feet per floor to get the number of needed floors.*

*Multiplying the number of floors by per floor gives a height of , which is approximately .*

1. Recall that every whole number is a finite decimal. [↑](#footnote-ref-1)
2. Sometimes the value of is known as the order of magnitude of the number , but we will use . [↑](#footnote-ref-2)