



Topic A:

Real Numbers

N-RN.A.1, N-RN.A.2, N-Q.A.2, F-IF.B.6, F-BF.A.1a, F-LE.A.2

Focus Standards:	N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)^3}$ to hold, so $\left(5^{\frac{1}{3}}\right)^3$ must equal 5.</i>
	N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
	N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.*
	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
	F-BF.A.1a	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context
	F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

Instructional Days: 6

- Lesson 1:** Integer Exponents (E)¹
- Lesson 2:** Base 10 and Scientific Notation (P)
- Lesson 3:** Rational Exponents—What are $2^{\frac{1}{2}}$ and $2^{\frac{1}{3}}$? (S)
- Lesson 4:** Properties of Exponents and Radicals (P)
- Lesson 5:** Irrational Exponents—What are $2^{\sqrt{2}}$ and 2^{π} ? (S)
- Lesson 6:** Euler's Number, e (P)

¹ Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson

In Topic A, students prepare to generalize what they know about various function families by examining the behavior of exponential functions. One goal of the module is to show that the domain of the exponential function, $f(x) = b^x$, where b is a positive number not equal to 1, is all real numbers. In Lesson 1, students review and practice applying the laws of exponents to expressions in which the exponents are integers. Students first tackle a challenge problem on paper folding that is related to exponential growth and then apply and practice applying the laws of exponents to rewriting algebraic expressions. They experiment, create a table of values, observe patterns, and then generalize a formula to represent different measurements in the folded stack of paper as specified in **F-LE.A.2**. They also use the laws of exponents to work with very large and very small numbers.

Lesson 2 sets the stage for the introduction of base 10 logarithms in Topic B of the module by reviewing how to express numbers using scientific notation, how to compute using scientific notation, and how to use the laws of exponents to simplify those computations in accordance with **N-RN.A.2**. Students should gain a sense of the change in magnitude when different powers of 10 are compared. The activities in these lessons prepare students for working with quantities that increase in magnitude by powers of 10 and by showing them the usefulness of exponent properties when performing arithmetic operations. Similar work will be done in later lessons relating to logarithms. Exercises on distances between planets in the solar system and on comparing magnitudes in other real-world contexts provide additional practice with arithmetic operations on numbers written using scientific notation.

Lesson 3 begins with students examining the graph of $y = 2^x$ and estimating the values as a means of extending their understanding of integer exponents to rational exponents. The examples are generalized to $2^{\frac{1}{n}}$ before generalizing further to $2^{\frac{m}{n}}$. As the domain of the identities involving exponents is expanded, it is important to maintain consistency with the properties already developed. Students work specifically to make sense that $2^{\frac{1}{2}} = \sqrt{2}$ and $2^{\frac{1}{3}} = \sqrt[3]{2}$ to develop the more general concept that $2^{\frac{1}{n}} = \sqrt[n]{2}$. The lesson demonstrates how people develop mathematics (a) to be consistent with what is already known and (b) to make additional progress. Additionally, students practice MP.7 as they extend the rules for integer exponents to rules for rational exponents (**N-RN.A.1**).

Lesson 4 continues the discussion of properties of exponents and radicals, and students continue to practice MP.7 as they extend their understanding of exponents to all rational numbers and for all positive real bases as specified in **N-RN.A.1**. Students rewrite expressions involving radicals and rational exponents using the properties of exponents (**N-RN.A.2**). The notation $x^{\frac{1}{n}}$ specifically indicates the principal root of x : the positive root when n is even and the real-valued root when n is odd. To avoid inconsistencies in our later work with logarithms, x is required to be positive.

Lesson 5 revisits the work of Lesson 3 and extends student understanding of the domain of the exponential function $f(x) = b^x$, where b is a positive real number, from the rational numbers to all real numbers through the process of considering what it means to raise a number to an irrational exponent (such as $2^{\sqrt{2}}$). In many ways, this lesson parallels the work students did in Lesson 3 to make a solid case for why the laws of exponents hold for all rational number exponents. The recursive procedure that students employ in this lesson aligns with **F-BF.A.1a**. This lesson is important both because it helps to portray mathematics as a coherent body of knowledge that makes sense and because we need to make sure that students understand that logarithms can be irrational numbers. Essentially, we need to guarantee that exponential and logarithmic functions are continuous functions. Students take away from these lessons an understanding that

the domain of exponents in the laws of exponents does indeed extend to all real numbers rather than integers, as defined previously in Grade 8.

Lesson 6 is a modeling lesson in which students practice MP.4 when they find an exponential function to model the amount of water in a tank after t seconds when the height of the water is constantly doubling or tripling, and apply **F-IF.B.6** as they explore the average rate of change of the height of the water over smaller and smaller intervals. If we denote the height of the water in the tank at time t seconds by $H(t) = b^t$, then the average rate of change of the height of the water on an interval $[T, T + \varepsilon]$ is approximated by

$\frac{H(T+\varepsilon)-H(T)}{\varepsilon} \approx c \cdot H(T)$. Students calculate that if the height of the water is doubling each second, then $c \approx 0.69$, and if the height of the water is tripling each second, then $c \approx 1.1$. Students discover Euler's number, e , by applying repeated reasoning (MP.8) and numerically approximating the base b for which the constant c is equal to 1. Euler's number will be used extensively in the future and occurs in many different applications.