Lesson 1: Integer Exponents

Classwork

Opening Exercise

Can you fold a piece of notebook paper in half times?

How thick will the folded paper be?

Will the area of the paper on the top of the folded stack be larger or smaller than a postage stamp?

Exploratory Challenge

* 1. What are the dimensions of your paper?
  2. How thick is one sheet of paper? Explain how you decided on your answer.
  3. Describe how you folded the paper.
  4. Record data in the following table based on the size and thickness of your paper.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number of Folds |  |  |  |  |  |  |  |  |  |  |  |
| Thickness of the Stack (in.) |  |  |  |  |  |  |  |  |  |  |  |
| Area of the Top of the Stack  (sq. in.) |  |  |  |  |  |  |  |  |  |  |  |

* 1. Were you able to fold a piece of notebook paper in half times? Why or why not?
  2. Create a formula that approximates the height of the stack after folds.
  3. Create a formula that will give you the approximate area of the top after folds.
  4. Answer the original questions from the Opening Exercise. How do the actual answers compare to your original predictions?

**Example 1: Using the Properties of Exponents to Rewrite Expressions**

The table below displays the thickness and area of a folded square sheet of gold foil. In 2001, Britney Gallivan, a California high school junior, successfully folded this size sheet of gold foil in half times to earn extra credit in her mathematics class.

Rewrite each of the table entries as a multiple of a power of .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of Folds | Thickness of the Stack  (Millionths of a Meter) | Thickness Using a Power of 2 | Area of the Top  (Square Inches) | Area  Using a Power of 2 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**Example 2: Applying the Properties of Exponents to Rewrite Expressions**

Rewrite each expression in the form of , where is a real number, is an integer, and is a nonzero real number.

Exercises 1–5

Rewrite each expression in the form of , where is a real number and is an integer. Assume .

Lesson Summary

The Properties of Exponents

For real numbers and with , , and all integers and , the following properties hold:

Problem Set

1. Suppose your class tried to fold an unrolled roll of toilet paper. It was originally wide and long. Toilet paper is approximately thick.
   1. Complete each table and represent the area and thickness using powers of.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of Folds | Thickness  After Folds  (inches) |  | Number of Folds | Area on Top  After Folds  (square inches) |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

* 1. Create an algebraic function that describes the area in square inches after folds.
  2. Create an algebraic function that describes the thickness in inches after folds.

1. In the Exit Ticket, we saw the formulas below. The first formula determines the minimum width, , of a square piece of paper of thickness needed to fold it in half times, alternating horizontal and vertical folds. The second formula determines the minimum length, , of a long rectangular piece of paper of thicknessneeded to fold it in half times, always folding perpendicular to the long side.

Use the appropriate formula to verify why it is possible to fold a inch by inch sheet of gold foil in half times. Use millionths of a meter for the thickness of gold foil.

1. Use the formula from the Exit Ticket to determine if you can fold an unrolled roll of toilet paper in half more than times. Assume that the thickness of a sheet of toilet paper is approximately and that one roll is long.
2. Apply the properties of exponents to rewrite expressions in the form , where is an integer and .
3. Apply the properties of exponents to verify that each statement is an identity.
   1. for integer values of .
   2. for integer values of
   3. for integer values of
4. Jonah was trying to rewrite expressions using the properties of exponents and properties of algebra for nonzero values of . In each problem, he made a mistake. Explain where he made a mistake in each part and provide a correct solution.

|  |
| --- |
| Jonah’s Incorrect Work |

1. If , and , express in terms of .
2. If , and , express in terms of .
3. If , and , show that .
4. Do the following without a calculator:
   1. Express as a power of .
   2. Divide by .
5. Use powers of to help you perform each calculation.
6. Write the first five terms of each of the following recursively-defined sequences:
   1. ,
   2. ,
   3. , , where is a real number. Write each term in the form .
   4. , , (). Write each term in the form .
7. In Module 1, you established the identity

, where is a real number and is a positive integer.

Use this identity to find explicit formulas as specified below.

* 1. Rewrite the given identity to isolate the sum for .
  2. Find an explicit formula for *.*
  3. Find an explicit formula for in terms of powers of .
  4. Jerry simplified the sum by writing . What did he do wrong?
  5. Find an explicit formula for in terms of powers of .
  6. Find an explicit formula for in terms of powers of . Hint: Use part (e).
  7. Find an explicit formula for in terms of powers .