Name ______________________________ Date ______________

Lesson 1: Integer Exponents

Exit Ticket

The following formulas for paper folding were discovered by Britney Gallivan in 2001 when she was a high school junior. The first formula determines the minimum width, W, of a square piece of paper of thickness T needed to fold it in half n times, alternating horizontal and vertical folds. The second formula determines the minimum length, L, of a long rectangular piece of paper of thickness T needed to fold it in half n times, always folding perpendicular to the long side.

$$W = \pi \cdot T \cdot 2^{\frac{3(n-1)}{2}} \qquad L = \frac{\pi T}{6}(2^n + 4)(2^n - 1)$$

1. Notebook paper is approximately 0.004 in. thick. Using the formula for the width W, determine how wide a square piece of notebook paper would need to be to successfully fold it in half 13 times, alternating horizontal and vertical folds.

2. Toilet paper is approximately 0.002 in. thick. Using the formula for the length L, how long would a continuous sheet of toilet paper have to be to fold it in half 12 times, folding perpendicular to the long edge each time?

3. Use the properties of exponents to rewrite each expression in the form kx^n. Then evaluate the expression for the given value of x.

   a. $2x^3 \cdot \frac{5}{4}x^{-1};\ x = 2$

   b. $\frac{9}{(2x)^{-3}};\ x = -\frac{1}{3}$

Name ______________________________ Date ______________

Lesson 2: Base 10 and Scientific Notation

Exit Ticket

1. A sheet of gold foil is 0.28 millionths of a meter thick. Write the thickness of a gold foil sheet measured in centimeters using scientific notation.

2. Without performing the calculation, estimate which expression is larger. Explain how you know.

$$(4 \times 10^{10})(2 \times 10^{5}) \qquad \text{and} \qquad \frac{4 \times 10^{12}}{2 \times 10^{-4}}$$

Name ____ Date ____________________

Lesson 3: Rational Exponents—What Are $2^{\frac{1}{2}}$ and $2^{\frac{1}{3}}$?

Exit Ticket

1. Write the following exponential expressions as equivalent radical expressions.

   a. $2^{\frac{1}{2}}$

   b. $2^{\frac{3}{4}}$

   c. $3^{-\frac{2}{3}}$

2. Rewrite the following radical expressions as equivalent exponential expressions.

   a. $\sqrt{5}$

   b. $2\sqrt[4]{3}$

   c. $\frac{1}{\sqrt[3]{16}}$

3. Provide a written explanation for each question below.

   a. Is it true that $\left(4^{\frac{1}{2}}\right)^3 = (4^3)^{\frac{1}{2}}$? Explain how you know.

   b. Is it true that $\left(1000^{\frac{1}{3}}\right)^3 = (1000^3)^{\frac{1}{3}}$? Explain how you know.

   c. Suppose that m and n are positive integers and b is a real number so that the principal n^{th} root of b exists. In general does $\left(b^{\frac{1}{n}}\right)^m = (b^m)^{\frac{1}{n}}$? Provide at least one example to support your claim.

Name ______________________________ Date ________________

Lesson 4: Properties of Exponents and Radicals

Exit Ticket

1. Find the exact value of $9^{\frac{11}{10}} \cdot 9^{\frac{2}{5}}$ without using a calculator.

2. Justify that $\sqrt[3]{8} \cdot \sqrt[3]{8} = \sqrt{16}$ using the properties of exponents in at least two different ways.

Name ______________________________ Date ______________

Lesson 5: Irrational Exponents—What Are $2^{\sqrt{2}}$ and 2^{π}?

Exit Ticket

Use the process outlined in the lesson to approximate the number $2^{\sqrt{3}}$. Use the approximation $\sqrt{3} \approx 1.7320508$.

a. Find a sequence of five intervals that contain $\sqrt{3}$ whose endpoints get successively closer to $\sqrt{3}$.

b. Find a sequence of five intervals that contain $2^{\sqrt{3}}$ whose endpoints get successively closer to $2^{\sqrt{3}}$. Write your intervals in the form $2^r < 2^{\sqrt{3}} < 2^s$ for rational numbers r and s.

c. Use your calculator to find approximations to four decimal places of the endpoints of the intervals in part (b).

d. Based on your work in part (c) what is your best estimate of the value of $2^{\sqrt{3}}$?

Name ____ Date ____________________

Lesson 6: Euler's Number, e

Exit Ticket

1. Suppose that water is entering a cylindrical water tank so that the initial height of the water is 3 cm and the height of the water doubles every 30 seconds. Write an equation of the height of the water at time t seconds.

2. Explain how the number e arose in our exploration of the average rate of change of the height of the water in the water tank.

Name ____ Date ____________________

Lesson 7: Bacteria and Exponential Growth

Exit Ticket

Loggerhead turtles reproduce every 2–4 years, laying approximately 120 eggs in a clutch. Using this information, we can derive an approximate equation to model the turtle population. As is often the case in biological studies, we will count only the female turtles. If we start with a population of one female turtle in a protected area, and assume that all turtles survive, we can roughly approximate the population of female turtles by $T(t) = 5^t$. Use the methods of the Example to find the number of years, Y, it will take for this model to predict that there will be 300 female turtles.

Name ____ Date ____________________

Lesson 8: The "WhatPower" Function

Exit Ticket

1. Explain why we need to specify $0 < b < 1$ and $b > 1$ as valid values for the base b in the expression $\log_b(x)$.

2. Calculate the following logarithms.
   a. $\log_5(25)$

   b. $\log_{10}\left(\frac{1}{100}\right)$

   c. $\log_9(3)$

Name ____ Date ____________________

Lesson 9: Logarithms—How Many Digits Do You Need?

Exit Ticket

A brand new school district needs to generate ID numbers for its student body. The district anticipates a total enrollment of $75{,}000$ students within the next ten years. Will a five-digit ID number comprising the symbols $0, 1, \ldots, 9$ be enough? Explain your reasoning.

Name ______________________________ Date ______________

Lesson 10: Building Logarithmic Tables

Exit Ticket

1. Use the log table below to approximate the following logarithms to four decimal places. Do not use a calculator.

x	$\log(x)$
1	0.0000
2	0.3010
3	0.4771
4	0.6021
5	0.6990

x	$\log(x)$
6	0.7782
7	0.8451
8	0.9031
9	0.9542
10	1.0000

a. $\log(500)$

b. $\log(0.0005)$

2. Suppose that A is a number with $\log(A) = 1.352$.

a. What is the value of $\log(1000A)$?

b. Which of the following is true? Explain how you know.

i. $A < 0$
ii. $0 < A < 10$
iii. $10 < A < 100$
iv. $100 < A < 1000$
v. $A > 1000$

Name ______________________________ Date ________________

Lesson 11: The Most Important Property of Logarithms

Exit Ticket

1. Use the table below to approximate the following logarithms to 4 decimal places. Do not use a calculator.

x	$\log(x)$
2	0.3010
3	0.4771
5	0.6990
7	0.8451

   a. $\log(9)$

   b. $\log\left(\frac{1}{15}\right)$

   c. $\log(45{,}000)$

2. Suppose that k is an integer, a is a positive real number, and you know the value of $\log(a)$. Explain how to find the value of $\log(10^k \cdot a^2)$.

Name ____ Date ____________________

Lesson 12: Properties of Logarithms

Exit Ticket

1. State as many of the six properties of logarithms as you can.

2. Use the properties of logarithms to show that $\log\left(\frac{1}{x}\right) = -\log(x)$ for all $x > 0$.

3. Use the properties of logarithms to show that $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$ for $x > 0$ and $y > 0$.

Name _______________________________ Date _______________

Lesson 13: Changing the Base

Exit Ticket

1. Are there any properties that hold for base 10 logarithms that would not be valid for the logarithm base e? Why? Are there any properties that hold for base 10 logarithms that would not be valid for some positive base b, such that $b \neq 1$?

2. Write each logarithm as an equivalent expression involving only logarithms base 10.
   a. $\log_3(25)$

   b. $\log_{100}(x^2)$

3. Rewrite each expression as an equivalent expression containing only one logarithm.
   a. $3\ln(p+q) - 2\ln(q) - 7\ln(p)$

   b. $\ln(xy) - \ln\left(\frac{x}{y}\right)$

Name ____ Date ____________________

Lesson 14: Solving Logarithmic Equations

Exit Ticket

Find all solutions to the following equations. Remember to check for extraneous solutions.

1. $5\log_2(3x+7)=0$

2. $\log(x-1)+\log(x-4)=1$

Name ____ Date ____________________

Lesson 15: Why Were Logarithms Developed?

Exit Ticket

The surface area of Jupiter is $6.14 \times 10^{10}\ \mathrm{km}^2$, and the surface area of Earth is $5.10 \times 10^{8}\ \mathrm{km}^2$. Without using a calculator but using the table of logarithms, find how many times greater the surface area of Jupiter is than the surface area of Earth.

Common Logarithm Table

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.0374
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0719	0.0755
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1072	0.1106
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1399	0.1430
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1703	0.1732
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1987	0.2014
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.2279
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.2529
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2742	0.2765
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2967	0.2989
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3181	0.3201
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3385	0.3404
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.3598
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.3784
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3945	0.3962
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4116	0.4133
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4281	0.4298
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4440	0.4456
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4594	0.4609
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4742	0.4757
3.0	0.4771	0.4786	0.4800	0.4814	0.4829	0.4843	0.4857	0.4871	0.4886	0.4900
3.1	0.4914	0.4928	0.4942	0.4955	0.4969	0.4983	0.4997	0.5011	0.5024	0.5038
3.2	0.5051	0.5065	0.5079	0.5092	0.5105	0.5119	0.5132	0.5145	0.5159	0.5172
3.3	0.5185	0.5198	0.5211	0.5224	0.5237	0.5250	0.5263	0.5276	0.5289	0.5302
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.5428
3.5	0.5441	0.5453	0.5465	0.5478	0.5490	0.5502	0.5514	0.5527	0.5539	0.5551
3.6	0.5563	0.5575	0.5587	0.5599	0.5611	0.5623	0.5635	0.5647	0.5658	0.5670
3.7	0.5682	0.5694	0.5705	0.5717	0.5729	0.5740	0.5752	0.5763	0.5775	0.5786
3.8	0.5798	0.5809	0.5821	0.5832	0.5843	0.5855	0.5866	0.5877	0.5888	0.5899
3.9	0.5911	0.5922	0.5933	0.5944	0.5955	0.5966	0.5977	0.5988	0.5999	0.6010
4.0	0.6021	0.6031	0.6042	0.6053	0.6064	0.6075	0.6085	0.6096	0.6107	0.6117
4.1	0.6128	0.6138	0.6149	0.6160	0.6170	0.6180	0.6191	0.6201	0.6212	0.6222
4.2	0.6232	0.6243	0.6253	0.6263	0.6274	0.6284	0.6294	0.6304	0.6314	0.6325
4.3	0.6335	0.6345	0.6355	0.6365	0.6375	0.6385	0.6395	0.6405	0.6415	0.6425
4.4	0.6435	0.6444	0.6454	0.6464	0.6474	0.6484	0.6493	0.6503	0.6513	0.6522
4.5	0.6532	0.6542	0.6551	0.6561	0.6571	0.6580	0.6590	0.6599	0.6609	0.6618
4.6	0.6628	0.6637	0.6646	0.6656	0.6665	0.6675	0.6684	0.6693	0.6702	0.6712
4.7	0.6721	0.6730	0.6739	0.6749	0.6758	0.6767	0.6776	0.6785	0.6794	0.6803
4.8	0.6812	0.6821	0.6830	0.6839	0.6848	0.6857	0.6866	0.6875	0.6884	0.6893
4.9	0.6902	0.6911	0.6920	0.6928	0.6937	0.6946	0.6955	0.6964	0.6972	0.6981
5.0	0.6990	0.6998	0.7007	0.7016	0.7024	0.7033	0.7042	0.7050	0.7059	0.7067
5.1	0.7076	0.7084	0.7093	0.7101	0.7110	0.7118	0.7126	0.7135	0.7143	0.7152
5.2	0.7160	0.7168	0.7177	0.7185	0.7193	0.7202	0.7210	0.7218	0.7226	0.7235
5.3	0.7243	0.7251	0.7259	0.7267	0.7275	0.7284	0.7292	0.7300	0.7308	0.7316
5.4	0.7324	0.7332	0.7340	0.7348	0.7356	0.7364	0.7372	0.7380	0.7388	0.7396

N	0	1	2	3	4	5	6	7	8	9
5.5	0.7404	0.7412	0.7419	0.7427	0.7435	0.7443	0.7451	0.7459	0.7466	0.7474
5.6	0.7482	0.7490	0.7497	0.7505	0.7513	0.7520	0.7528	0.7536	0.7543	0.7551
5.7	0.7559	0.7566	0.7574	0.7582	0.7589	0.7597	0.7604	0.7612	0.7619	0.7627
5.8	0.7634	0.7642	0.7649	0.7657	0.7664	0.7672	0.7679	0.7686	0.7694	0.7701
5.9	0.7709	0.7716	0.7723	0.7731	0.7738	0.7745	0.7752	0.7760	0.7767	0.7774
6.0	0.7782	0.7789	0.7796	0.7803	0.7810	0.7818	0.7825	0.7832	0.7839	0.7846
6.1	0.7853	0.7860	0.7868	0.7875	0.7882	0.7889	0.7896	0.7903	0.7910	0.7917
6.2	0.7924	0.7931	0.7938	0.7945	0.7952	0.7959	0.7966	0.7973	0.7980	0.7987
6.3	0.7993	0.8000	0.8007	0.8014	0.8021	0.8028	0.8035	0.8041	0.8048	0.8055
6.4	0.8062	0.8069	0.8075	0.8082	0.8089	0.8096	0.8102	0.8109	0.8116	0.8122
6.5	0.8129	0.8136	0.8142	0.8149	0.8156	0.8162	0.8169	0.8176	0.8182	0.8189
6.6	0.8195	0.8202	0.8209	0.8215	0.8222	0.8228	0.8235	0.8241	0.8248	0.8254
6.7	0.8261	0.8267	0.8274	0.8280	0.8287	0.8293	0.8299	0.8306	0.8312	0.8319
6.8	0.8325	0.8331	0.8338	0.8344	0.8351	0.8357	0.8363	0.8370	0.8376	0.8382
6.9	0.8388	0.8395	0.8401	0.8407	0.8414	0.8420	0.8426	0.8432	0.8439	0.8445
7.0	0.8451	0.8457	0.8463	0.8470	0.8476	0.8482	0.8488	0.8494	0.8500	0.8506
7.1	0.8513	0.8519	0.8525	0.8531	0.8537	0.8543	0.8549	0.8555	0.8561	0.8567
7.2	0.8573	0.8579	0.8585	0.8591	0.8597	0.8603	0.8609	0.8615	0.8621	0.8627
7.3	0.8633	0.8639	0.8645	0.8651	0.8657	0.8663	0.8669	0.8675	0.8681	0.8686
7.4	0.8692	0.8698	0.8704	0.8710	0.8716	0.8722	0.8727	0.8733	0.8739	0.8745
7.5	0.8751	0.8756	0.8762	0.8768	0.8774	0.8779	0.8785	0.8791	0.8797	0.8802
7.6	0.8808	0.8814	0.8820	0.8825	0.8831	0.8837	0.8842	0.8848	0.8854	0.8859
7.7	0.8865	0.8871	0.8876	0.8882	0.8887	0.8893	0.8899	0.8904	0.8910	0.8915
7.8	0.8921	0.8927	0.8932	0.8938	0.8943	0.8949	0.8954	0.8960	0.8965	0.8971
7.9	0.8976	0.8982	0.8987	0.8993	0.8998	0.9004	0.9009	0.9015	0.9020	0.9025
8.0	0.9031	0.9036	0.9042	0.9047	0.9053	0.9058	0.9063	0.9069	0.9074	0.9079
8.1	0.9085	0.9090	0.9096	0.9101	0.9106	0.9112	0.9117	0.9122	0.9128	0.9133
8.2	0.9138	0.9143	0.9149	0.9154	0.9159	0.9165	0.9170	0.9175	0.9180	0.9186
8.3	0.9191	0.9196	0.9201	0.9206	0.9212	0.9217	0.9222	0.9227	0.9232	0.9238
8.4	0.9243	0.9248	0.9253	0.9258	0.9263	0.9269	0.9274	0.9279	0.9284	0.9289
8.5	0.9294	0.9299	0.9304	0.9309	0.9315	0.9320	0.9325	0.9330	0.9335	0.9340
8.6	0.9345	0.9350	0.9355	0.9360	0.9365	0.9370	0.9375	0.9380	0.9385	0.9390
8.7	0.9395	0.9400	0.9405	0.9410	0.9415	0.9420	0.9425	0.9430	0.9435	0.9440
8.8	0.9445	0.9450	0.9455	0.9460	0.9465	0.9469	0.9474	0.9479	0.9484	0.9489
8.9	0.9494	0.9499	0.9504	0.9509	0.9513	0.9518	0.9523	0.9528	0.9533	0.9538
9.0	0.9542	0.9547	0.9552	0.9557	0.9562	0.9566	0.9571	0.9576	0.9581	0.9586
9.1	0.9590	0.9595	0.9600	0.9605	0.9609	0.9614	0.9619	0.9624	0.9628	0.9633
9.2	0.9638	0.9643	0.9647	0.9652	0.9657	0.9661	0.9666	0.9671	0.9675	0.9680
9.3	0.9685	0.9689	0.9694	0.9699	0.9703	0.9708	0.9713	0.9717	0.9722	0.9727
9.4	0.9731	0.9736	0.9741	0.9745	0.9750	0.9754	0.9759	0.9763	0.9768	0.9773
9.5	0.9777	0.9782	0.9786	0.9791	0.9795	0.9800	0.9805	0.9809	0.9814	0.9818
9.6	0.9823	0.9827	0.9832	0.9836	0.9841	0.9845	0.9850	0.9854	0.9859	0.9863
9.7	0.9868	0.9872	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	0.9908
9.8	0.9912	0.9917	0.9921	0.9926	0.9930	0.9934	0.9939	0.9943	0.9948	0.9952
9.9	0.9956	0.9961	0.9965	0.9969	0.9974	0.9978	0.9983	0.9987	0.9991	0.9996

Name ____ Date ____________________

1. Use properties of exponents to explain why it makes sense to define $16^{\frac{1}{4}}$ as $\sqrt[4]{16}$.

2. Use properties of exponents to rewrite each expression as either an integer or as a quotient of integers $\frac{p}{q}$ to show the expression is a rational number.

   a. $\sqrt[4]{2}\,\sqrt[4]{8}$

   b. $\dfrac{\sqrt[3]{54}}{\sqrt[3]{2}}$

   c. $16^{\frac{3}{2}} \cdot \left(\frac{1}{27}\right)^{\frac{2}{3}}$

3. Use properties of exponents to rewrite each expression with only positive, rational exponents. Then find the numerical value of each expression when $x = 9$, $y = 8$, and $z = 16$. In each case, the expression evaluates to a rational number.

   a. $\sqrt{\dfrac{xy^2}{(x^3z)^{\frac{1}{2}}}}$

   b. $\sqrt[11]{y^2z^4}$

   c. $x^{-\frac{3}{2}}y^{\frac{4}{3}}z^{-\frac{3}{4}}$

4. We can use finite approximations of the decimal expansion of $\pi = 3.141519 \ldots$ to find an approximate value of the number 3^{π}.

   a. Fill in the missing exponents in the following sequence of inequalities that represents the recursive process of finding the value of 3^{π}.

$$3^{3} < 3^{\pi} < 3^{4}$$
$$3^{3.1} < 3^{\pi} < 3^{3.2}$$
$$3^{3.14} < 3^{\pi} < 3^{3.15}$$
$$3^{(\quad)} < 3^{\pi} < 3^{(\quad)}$$
$$3^{(\quad\ )} < 3^{\pi} < 3^{(\quad\ )}$$

   b. Explain how this recursive process leads to better and better approximations of the number 3^{π}.

5. A scientist is studying the growth of a population of bacteria. At the beginning of her study, she has 800 bacteria. She notices that the population is quadrupling every hour.

   a. What quantities, including units, need to be identified to further investigate the growth of this bacteria population?

b. The scientist recorded the following information in her notebook, but she forgot to label each row. Label each row to show what quantities, including appropriate units, are represented by the numbers in the table, and then complete the table.

	0	1	2	3	4
	8	32	128		

c. Write an explicit formula for the number of bacteria present after t hours.

d. Another scientist studying the same population notices that the population is doubling every half an hour. Complete the table, and write an explicit formula for the number of bacteria present after x half hours.

Time, t (hours)	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
Time, x (half-hours)	0	1	2	3	4	5	6
Bacteria (hundreds)	8	16	32				

e. Find the time, in hours, when there will be $5{,}120{,}000$ bacteria. Express your answer as a logarithmic expression.

f. A scientist calculated the average rate of change for the bacteria in the first three hours to be 168. Which units should the scientist use when reporting this number?

6. Solve each equation. Express your answer as a logarithm, and then approximate the solution to the nearest thousandth.

   a. $3(10)^{-x} = \frac{1}{9}$

   b. $362\left(10^{\frac{t}{12}}\right) = 500$

   c. $(2)^{3x} = 9$

   d. $300e^{0.4t} = 900$

7. Because atoms and molecules are very small, they are counted in units of *moles*, where $1 \text{ mole} = 6.022 \times 10^{23}$. Concentration of molecules in a liquid is measured in units of moles per liter. The measure of the acidity of a liquid is called the pH of the liquid and is given by the formula

$$\text{pH} = -\log(H),$$

where H is the concentration of hydrogen ions in units of moles per liter.

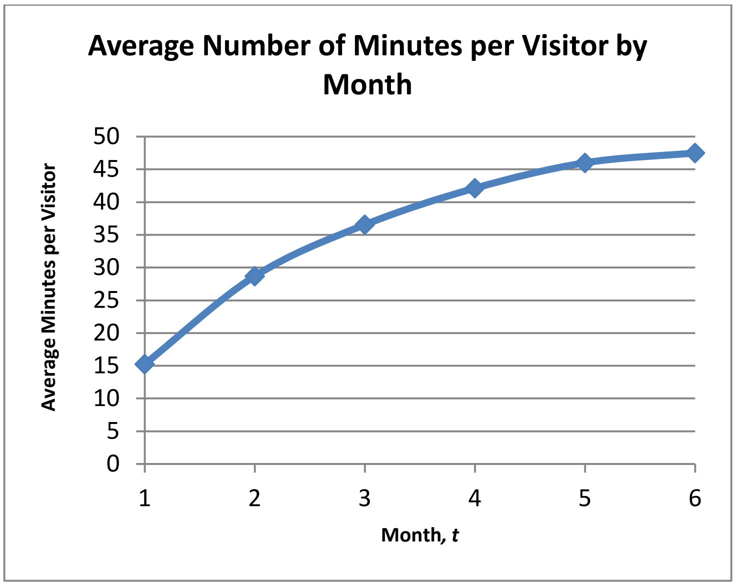
a. Water has a pH value of 7.0. How many hydrogen ions are in one liter of water?

b. If a liquid has a pH value larger than 7.0, does one liter of that liquid contain more or less hydrogen ions than one liter of water? Explain.

c. Suppose that liquid A is more acidic than liquid B, and their pH values differ by 1.2. What is the ratio of the concentration of hydrogen ions in liquid A to the concentration of hydrogen ions in liquid B?

8. A social media site is experiencing rapid growth. The table below shows values of V, the total number of unique visitors in each month, t, for a 6-month period of time. The graph shows the average minutes per visit to the site, M, in each month, t, for the same 6-month period of time.

t, Month	1	2	3	4	5	6
$V(t)$, Number of Unique Visitors	418,000	608,000	1,031,000	1,270,000	2,023,000	3,295,000



a. Between which two months did the site experience the most growth in total unique visitors? What is the average rate of change over this time interval?

b. Compute the value of $\frac{V(6)-V(1)}{6-1}$, and explain its meaning in this situation.

c. Between which two months did the average length of a visit change by the least amount? Estimate the average rate of change over this time interval.

d. Estimate the value of $\frac{M(3)-M(2)}{3-2}$ from the graph of M, and explain its meaning in this situation.

e. Based on the patterns they see in the table, the company predicts that the number of unique visitors will double each month after the sixth month. If growth continues at this pace, when will the number of unique visitors reach 1 billion?

Name ____ Date ____________________

Lesson 16: Rational and Irrational Numbers

Exit Ticket

The decimal expansion of e and $\sqrt{5}$ are given below.

$$e \approx 2.71828182\ldots$$

$$\sqrt{5} \approx 2.23606797\ldots$$

a. Find an approximation of $\sqrt{5}+e$ to three decimal places. Do not use a calculator.

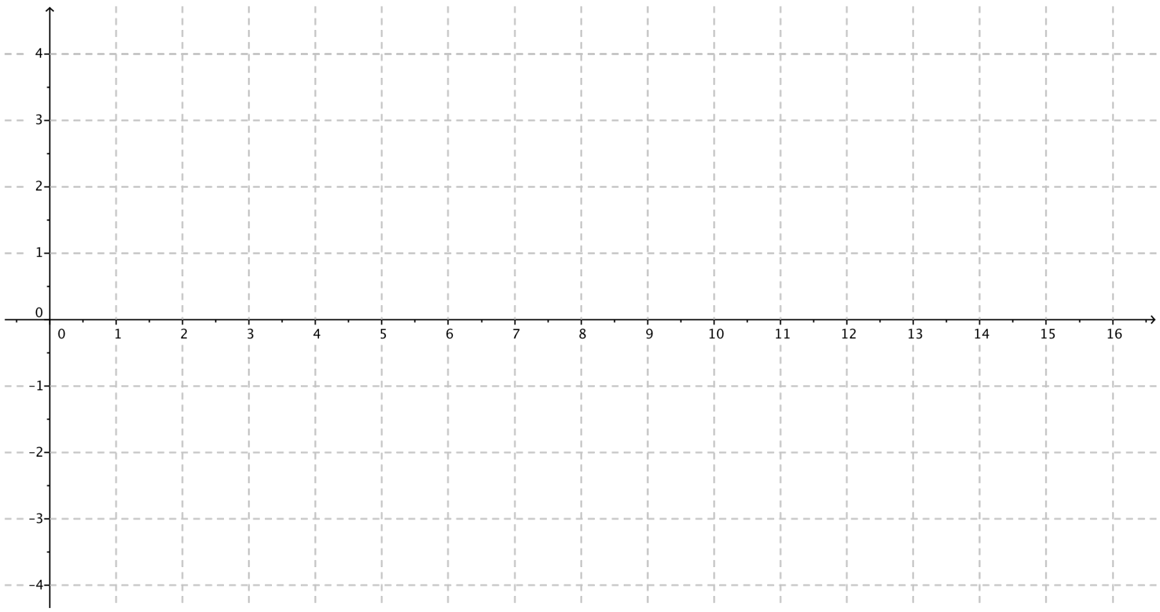
b. Explain how you can locate $\sqrt{5}+e$ on the number line. How is this different from locating $2.6+2.7$ on the number line?

Name ____ Date ____________________

Lesson 17: Graphing the Logarithm Function

Exit Ticket

Graph the function $f(x) = \log_3(x)$ without using a calculator, and identify its key features.

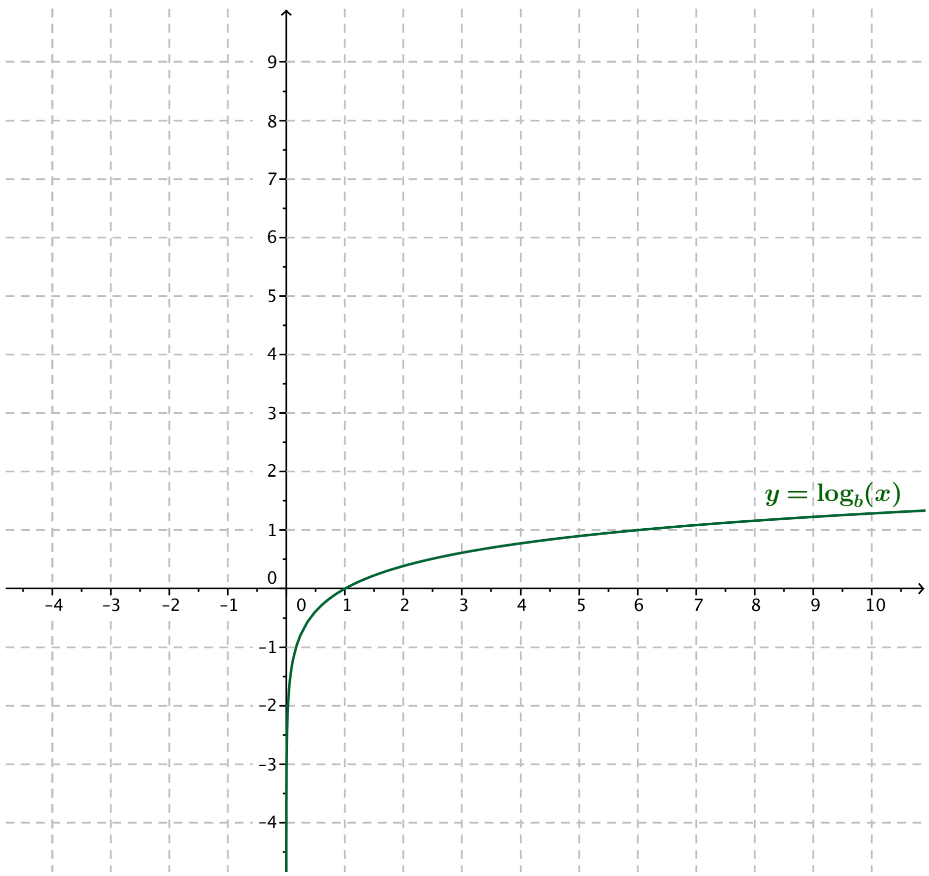


Name ____ Date ____________________

Lesson 18: Graphs of Exponential Functions and Logarithmic Functions

Exit Ticket

The graph of a logarithmic function $g(x) = \log_b(x)$ is shown below.



a. Explain how to find points on the graph of the function $f(x) = b^x$.

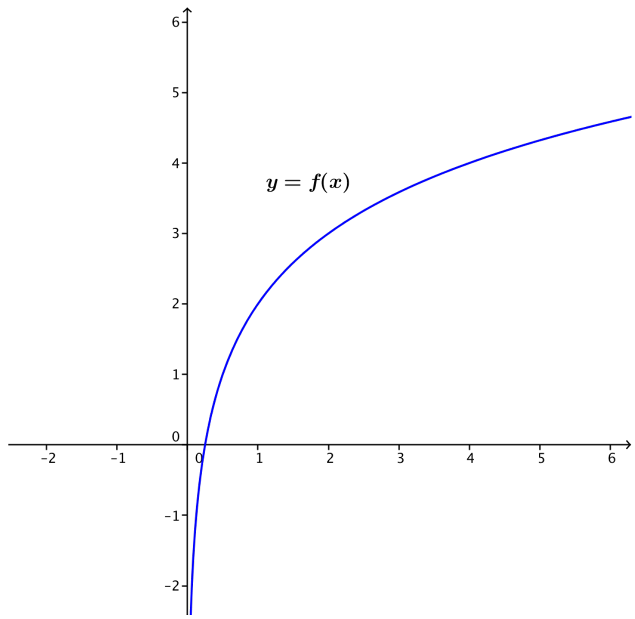
b. Sketch the graph of the function $f(x) = b^x$ on the same axes.

Name ____ Date ____________________

Lesson 19: The Inverse Relationship Between Logarithmic and Exponential Functions

Exit Ticket

1. The graph of a function f is shown below. Sketch the graph of its inverse function g on the same axes.



2. Explain how you made your sketch.

3. The function f graphed above is the function $f(x) = \log_2(x) + 2$ for $x > 0$. Find a formula for the inverse of this function.

Name ______________________________ Date ________________

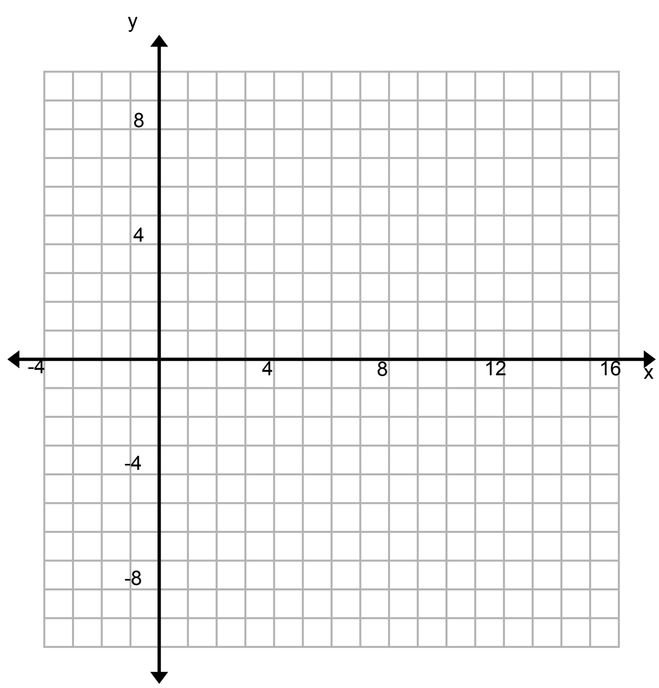
Lesson 20: Transformations of the Graphs of Logarithmic and Exponential Functions

Exit Ticket

1. Express $g(x) = -\log_4(2x)$ in the general form of a logarithmic function, $f(x) = k + a\log_b(x - h)$. Identify a, b, h, and k.

2. Use the structure of g when written in general from to describe the graph of g as a transformation of the graph of $h(x) = \log_4(x)$.

3. Graph g anad h on the same coordinate axes.



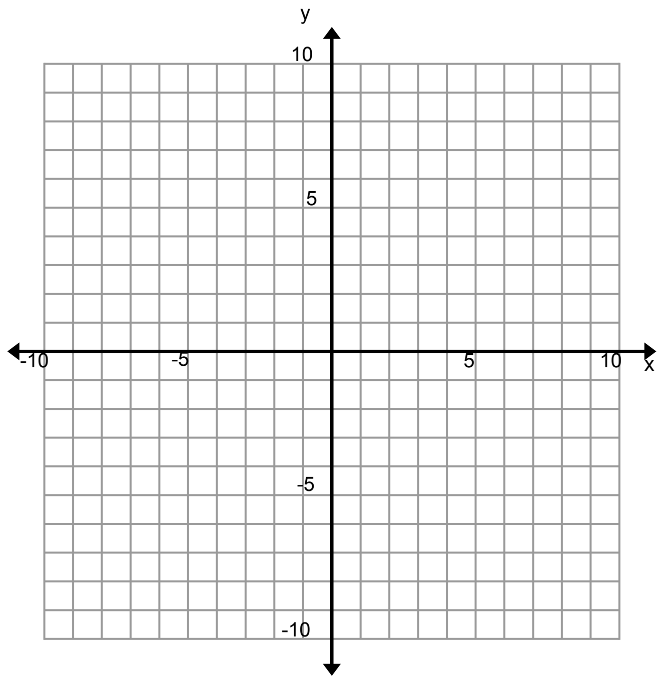
Name ____ Date ____________________

Lesson 21: The Graph of the Natural Logarithm Function

Exit Ticket

1. Describe the graph of $g(x) = 2 - \ln(x + 3)$ as a transformation of the graph of $f(x) = \ln(x)$.

2. Sketch the graphs of f and g by hand.



3. Explain where the graph of $g(x) = \log_3(2x)$ would sit in relation to the graph of $f(x) = \ln(x)$. Justify your answer using properties of logarithms and your knowledge of transformations of graph of functions.

Name ______________________________ Date ______________

Lesson 22: Choosing a Model

Exit Ticket

The amount of caffeine in a patient's bloodstream decreases by half every 3.5 hours. A latte contains 150 mg of caffeine, which is absorbed into the bloodstream almost immediately.

a. What type of function models the caffeine level in the patient's bloodstream at time t hours after drinking the latte? Explain how you know.

b. Do you have enough information to find a model that is appropriate for this situation? Either find a model or explain what other information you would need to do so.

Name ____ Date ____________________

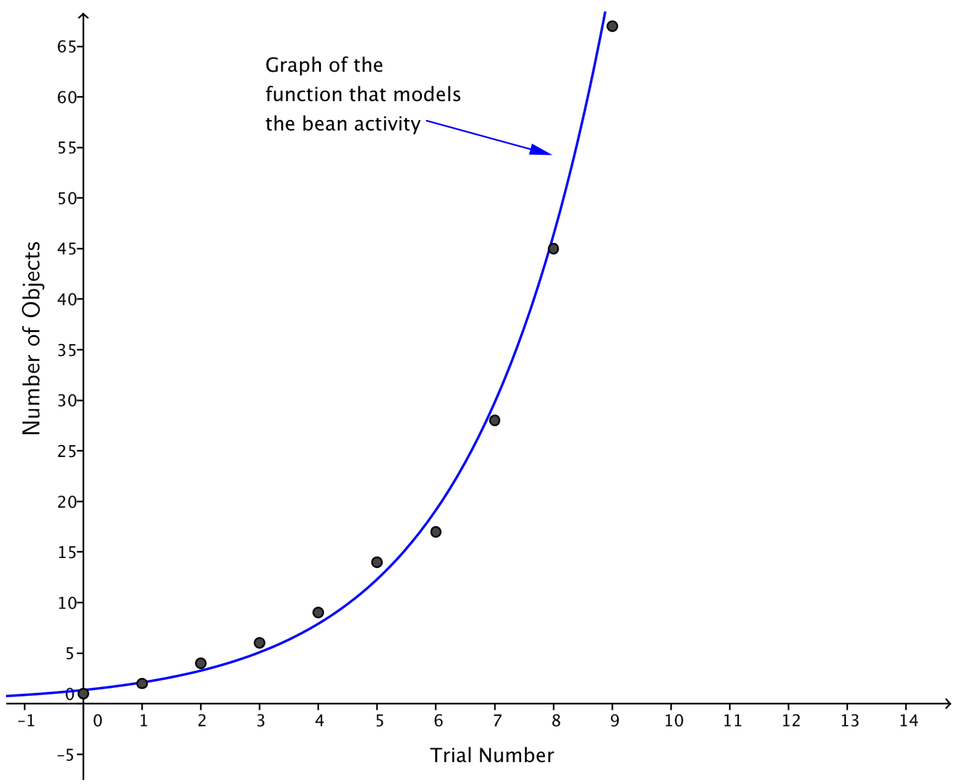
Lesson 23: Bean Counting

Exit Ticket

Suppose that you were to repeat the bean activity, but in place of beans, you were to use six-sided dice. Starting with one die, each time a die is rolled with a 6 showing, you add a new die to your cup.

a. Would the number of dice in your cup grow more quickly or more slowly than the number of beans did? Explain how you know.

b. A sketch of one sample of data from the bean activity is shown below. On the same axes, draw a rough sketch of how you would expect the line of best fit from the dice activity to look.



Name ______________________________ Date ________________

Lesson 24: Solving Exponential Equations

Exit Ticket

Consider the functions $f(x) = 2^{x+6}$ and $g(x) = 5^{2x}$.

a. Use properties of logarithms to solve the equation $f(x) = g(x)$. Give your answer as a logarithmic expression, and approximate it to two decimal places.

b. Verify your answer by graphing the functions $y = f(x)$ and $y = g(x)$ in the same window on a calculator, and sketch your graphs below. Explain how the graph validates your solution to part (a).

Name ____ Date ____________________

Lesson 25: Geometric Sequences and Exponential Growth and Decay

Exit Ticket

1. Every year, Mikhail receives a 3% raise in his annual salary. His starting annual salary was $\$40{,}000$.
   a. Does a geometric or arithmetic sequence best model Mikhail's salary in year n? Explain how you know.

   b. Find a recursive formula for a sequence, S_n, which represents Mikhail's salary in year n.

2. Carmela's annual salary in year n can be modeled by the recursive sequence $C_{n+1} = 1.05\, C_n$, where $C_0 = \$75{,}000$.
   a. What does the number 1.05 represent in the context of this problem?

   b. What does the number $\$75{,}000$ represent in the context of this problem?

   c. Find an explicit formula for a sequence that represents Carmela's salary.

Name ____ Date ____________________

Lesson 26: Percent Rate of Change

Exit Ticket

April would like to invest $\$200$ in the bank for one year. Three banks all have a nominal APR of 1.5%, but compound the interest differently.

a. Bank A computes interest just once at the end of the year. What would April's balance be after one year with this bank?

b. Bank B compounds interest at the end of each six-month period. What would April's balance be after one year with this bank?

c. Bank C compounds interest continuously. What would April's balance be after one year with this bank?

d. Each bank decides to double the nominal APR it offers for one year. That is, they offer a nominal APR of 3%. Each bank advertises, "DOUBLE THE AMOUNT YOU EARN!" For which of the three banks, if any, is this advertised claim correct?

Name ____ Date ____________________

Lesson 27: Modeling with Exponential Functions

Exit Ticket

1. The table below gives the average annual cost (e.g., tuition, room, and board) for four-year public colleges and universities. Explain why a linear model might not be appropriate for this situation.

Year	Average Annual Cost
1981	$\$2,550$
1991	$\$5,243$
2001	$\$8,653$
2011	$\$15,918$

2. Algebraically determine an exponential function to model this situation.

3. Use the properties of exponents to rewrite the function from Problem 2 to determine an annual growth rate.

4. If this trend continues, when will the average annual cost of attendance exceed $\$35,000$?

Name ______________________________ Date ______________

Lesson 28: Newton's Law of Cooling, Revisited

Exit Ticket

A pizza, heated to a temperature of $400°\text{F}$, is taken out of an oven and placed in a $75°\text{F}$ room at time $t = 0$ minutes. The temperature of the pizza is changing such that its decay constant, k, is 0.325. At what time is the temperature of the pizza $150°\text{F}$ and, therefore, safe to eat? Give your answer in minutes.

Name ____ Date ____________________

Lesson 29: The Mathematics Behind a Structured Savings Plan

Exit Ticket

Martin attends a financial planning conference and creates a budget for himself, realizing that he can afford to put away $\$200$ every month in savings and that he should be able to keep this up for two years. If Martin has the choice between an account earning an interest rate of 2.3% yearly versus an account earning an annual interest rate of 2.125% compounded monthly, which account will give Martin the largest return in two years?

Name ____ Date ________________

Lesson 30: Buying a Car

Exit Ticket

Fran wants to purchase a new boat. She starts looking for a boat around $\$6{,}000$. Fran creates a budget and thinks that she can afford $\$250$ every month for 2 years. Her bank charges her 5% interest per year, compounded monthly.

1. What is the actual monthly payment for Fran's loan?

2. If Fran can only pay $\$250$ per month, what is the most expensive boat she can buy without a down payment?

Name ____ Date ____________________

Lesson 31: Credit Cards

Exit Ticket

Suppose that you currently have one credit card with a balance of $\$10{,}000$ at an annual rate of 24.00% interest. You have stopped adding any additional charges to this card and are determined to pay off the balance. You have worked out the formula $b_n = b_0 r^n - R(1 + r + r^2 + \cdots + r^{n-1})$, where b_0 is the initial balance, b_n is the balance after you have made n payments, $r = 1 + i$, where i is the monthly interest rate, and R is the amount you are planning to pay each month.

a. What is the monthly interest rate i? What is the growth rate, r?

b. Explain why we can rewrite the given formula as $b_n = b_0 r^n - R\left(\frac{1-r^n}{1-r}\right)$.

c. How long will it take you to pay off this debt if you can afford to pay a constant $\$250$ per month? Give your answer in years and months.

Name ____ Date ____________________

Lesson 32: Buying a House

Exit Ticket

1. Recall the present value of an annuity formula, where A_p is the present value, R is the monthly payment, i is the monthly interest rate, and n is the number of monthly payments:

$$A_p = R\left(\frac{1-(1+i)^{-n}}{i}\right).$$

Rewrite this formula to isolate R.

2. Suppose that you want to buy a house that costs $\$175{,}000$. You can make a 10% down payment, and 1.2% of the house's value is paid into the escrow account each month.

    a. Find the monthly payment for a 30-year mortgage on this house.

    b. Find the monthly payment for a 15-year mortgage on this house.

Name ____ Date ____________________

Lesson 33: The Million Dollar Problem

Exit Ticket

1. At age 25, you begin planning for retirement at 65. Knowing that you have 40 years to save up for retirement and expecting an interest rate of 4% per year compounded monthly throughout the 40 years, how much do you need to deposit every month to save up $\$2$ million for retirement?

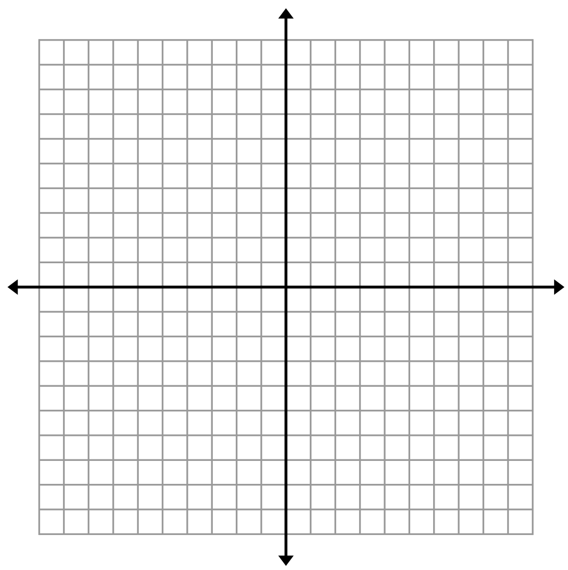
2. Currently, your savings for each month is capped at $\$400$. If you start investing all of this into a savings plan earning 1% interest annually, compounded monthly, then how long will it take to save $\$160{,}000$? (Hint: Use logarithms.)

Name ____ Date ________________________

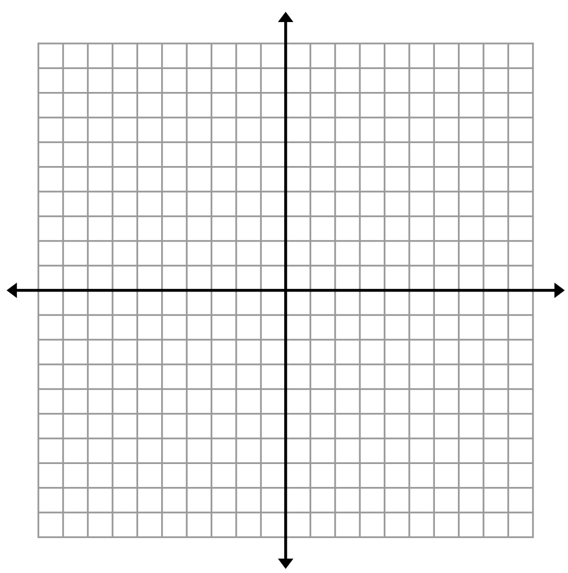
1. For parts (a) to (c),

   - Sketch the graph of each pair of functions on the same coordinate axes showing end behavior and intercepts, and
   - Describe the graph of g as a series of transformations of the graph of f.

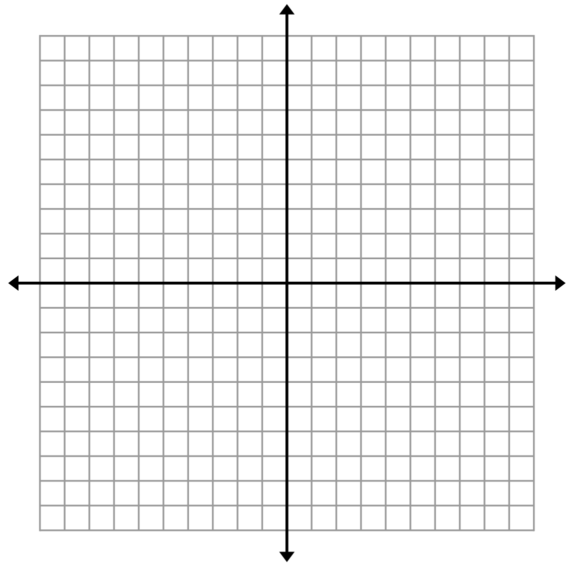
   a. $f(x) = 2^x$, and $g(x) = 2^{-x} + 3$



   b. $f(x) = 3^x$, and $g(x) = 9^{x-2}$



c. $f(x) = \log_2(x)$, and $g(x) = \log_2(x-1)^2$

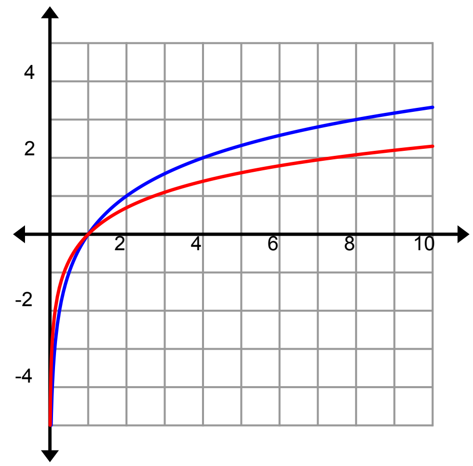


2. Consider the graph of $f(x) = 8^x$. Let $g(x) = f\left(\frac{1}{3}x + \frac{2}{3}\right)$ and $h(x) = 4f\left(\frac{x}{3}\right)$.

a. Describe the graphs of g and h as transformations of the graph of f.

b. Use the properties of exponents to show why the graphs of the functions $g(x) = f\left(\frac{1}{3}x + \frac{2}{3}\right)$ and $h(x) = 4f\left(\frac{x}{3}\right)$ are the same.

3. The graphs of the functions $f(x) = \ln(x)$ and $g(x) = \log_2(x)$ are shown to the right.



   a. Which curve is the graph of f, and which curve is the graph of g? Explain.

   b. Describe the graph of g as a transformation of the graph of f.

   c. By what factor has the graph of f been scaled vertically to produce the graph of g? Explain how you know.

4. Gwyneth is conducting an experiment. She rolls $1{,}000$ dice simultaneously and removes any that have a six showing. She then rerolls all of the dice that remain and again removes any that have a six showing. Gwyneth does this over and over again—rerolling the remaining dice and then removing those that land with a six showing.

   a. Write an exponential function f of the form $f(n) = a \cdot b^{cn}$ for any real number $n \geq 0$ that could be used to model the average number of dice she could expect on the n^{th} roll if she ran her experiment a large number of times.

   b. Gwyneth computed $f(12) = 112.15\ldots$ using the function f. How should she interpret the number $112.15\ldots$ in the context of the experiment?

   c. Explain the meaning of the parameters in your function f in terms of this experiment.

   d. Describe in words the key features of the graph of the function f for $n \geq 0$. Be sure to describe where the function is increasing or decreasing, where it has maximums and minimums (if they exist), and the end behavior.

e. According to the model, on which roll does Gwyneth expect, on average, to find herself with only one die remaining? Write and solve an equation to support your answer to this question.

f. For all of the values in the domain of f, is there any value for which f will predict an average number of 0 dice remaining? Explain why or why not. Be sure to use the domain of the function and the graph to support your reasoning.

Suppose the table below represents the results of one trial of Gwyneth's experiment.

Roll	Number of Dice Left	Roll	Number of Dice Left	Roll	Number of Dice Left
0	1000	10	157	20	26
1	840	11	139	21	22
2	692	12	115	22	15
3	581	13	90	23	13
4	475	14	78	24	10
5	400	15	63	25	6
6	341	16	55	26	2
7	282	17	43	27	1
8	232	18	40	28	0
9	190	19	33		

g. Let g be the function that is defined exactly by the data in the table, i.e., $g(0) = 1000$, $g(1) = 840$, $g(2) = 692$, and so forth, up to $g(28) = 0$. Describe in words how the graph of g looks different from the graph of f. Be sure to use the domain of g and the domain of f to justify your description.

h. Gwyneth runs her experiment hundreds of times, and each time she generates a table like the one above. How are these tables similar to the function f? How are they different?

5. Find the inverse g for each function f.

a. $f(x) = \frac{1}{2}x - 3$

b. $f(x) = \frac{x+3}{x-2}$

c. $f(x) = 2^{3x} + 1$

d. $f(x) = e^{x-3}$

e. $f(x) = \log(2x + 3)$

6. Dani has $\$1{,}000$ in an investment account that earns 3% per year, compounded monthly.

   a. Write a recursive sequence for the amount of money in her account after n months.

   b. Write an explicit formula for the amount of money in the account after n months.

   c. Write an explicit formula for the amount of money in her account after t years.

   d. Boris also has $\$1{,}000$, but in an account that earns 3% per year, compounded yearly. Write an explicit formula for the amount of money in his account after t years.

   e. Boris claims that the equivalent monthly interest rate for his account would be the same as Dani's. Use the expression you wrote in part (d) and the properties of exponents to show why Boris is incorrect.

7. Show that

$$\sum_{k=0}^{n} a \cdot r^k = a\left(\frac{1-r^n}{1-r}\right)$$

where $r \neq 1$.

8. Sami opens an account and deposits $\$100$ into it at the end of each month. The account earns 2% per year compounded monthly. Let $S(n)$ denote the amount of money in her account at the end of n months (just after she makes a deposit). For example, $S(1) = 100$ and $S(2) = 100\left(1 + \frac{0.02}{12}\right) + 100$.

   a. Write a geometric series for the amount of money in the account after 3, 4, and 5 months.

   b. Find a recursive description for $S(n)$.

c. Find an explicit function for $S(n)$, and use it to find $S(12)$.

d. When will Sami have at least $\$5{,}000$ in her account? Show work to support your answer.

9. Beatrice decides to deposit $\$100$ per month at the end of every month in a bank with an annual interest rate of 5.5% compounded monthly.

a. Write a geometric series to show how much she will accumulate in her account after one year.

b. Use the formula for the sum of a geometric series to calculate how much she will have in the bank after five years if she keeps on investing $\$100$ per month.

10. Nina has just taken out a car loan for $\$12{,}000$. She will pay an annual interest rate of 3% through a series of monthly payments for 60 months, which she pays at the end of each month. The amount of money she has left to pay on the loan at the end of the n^{th} month can be modeled by the function $f(n) = 86248 - 74248(1.0025)^n$ for $0 \leq n \leq 60$.

    At the same time as her first payment (at the end of the first month), Nina placed $\$100$ into a separate investment account that earns 6% per year compounded monthly. She placed $\$100$ into the account at the end of each month thereafter. The amount of money in her savings account at the end of the n^{th} month can be modeled by the function $g(n) = 20000(1.005)^n - 20000$ for $n \geq 0$.

    a. Use the functions f and g to write an equation whose solution could be used to determine when Nina will have saved enough money to pay off the remaining balance on her car loan.

    b. Use a calculator or computer to graph f and g on the same coordinate plane. Sketch the graphs below, labeling intercepts and indicating end behavior on the sketch. Include the coordinates of any intersection points.

c. How would you interpret the end behavior of each function in the context of this situation?

d. What does the intersection point mean in the context of this situation? Explain how you know.

e. After how many months will Nina have enough money saved to pay off her car loan? Explain how you know.

11. Each function below models the growth of three different trees of different ages over a fixed time interval.

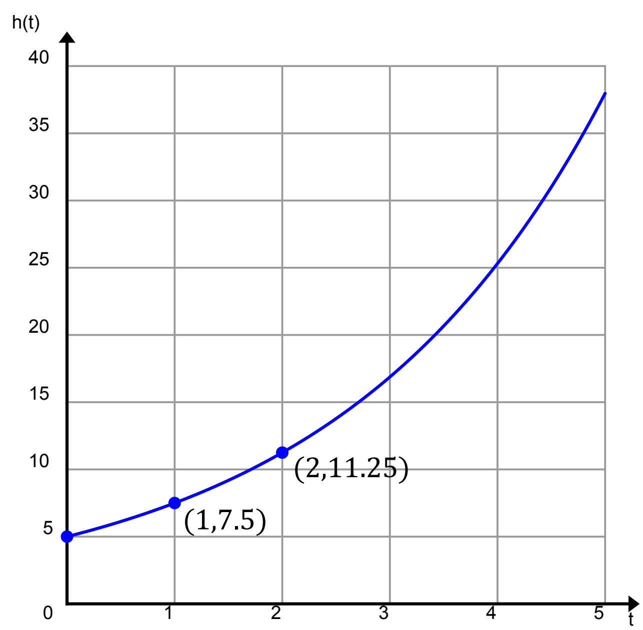
Tree A:

$f(t) = 15(1.69)^{\frac{t}{2}}$, where t is time in years since the tree was 15 feet tall, $f(t)$ is the height of the tree in feet, and $0 \le t \le 4$.

Tree B:

Years since the tree was 5 feet tall, t	Height in feet after t years, $g(t)$
0	5
1	6.3
2	7.6
3	8.9
4	10.2

Tree C: The graph of h is shown where t is years since the tree was 5 feet tall, and $h(t)$ is the height in feet after t years.



a. Classify each function f and g as representing a linear or nonlinear function. Justify your answers.

b. Use the properties of exponents to show that Tree A has a percent rate of change of 30% per year.

c. Which tree, A or C, has the greatest percent rate of change? Justify your answer.

d. Which function has the greatest average rate of change over the interval $[0,4]$, and what does that mean in terms of tree heights?

e. Write formulas for functions g and h, and use them to confirm your answer to part (c).

f. For the exponential models, if the average rate of change of one function over the interval $[0,4]$ is greater than the average rate of change of another function on the same interval, is the percent rate of change also greater? Why or why not?

12. Identify which functions are exponential. For the exponential functions, use the properties of exponents to identify the percent rate of change, and classify them as representing exponential growth or decay.

a. $f(x) = 3(1 - 0.4)^{-x}$

b. $g(x) = \frac{3}{4^x}$

c. $k(x) = 3x^{0.4}$

d. $h(x) = 3^{\frac{x}{4}}$

13. A patient in a hospital needs to maintain a certain amount of a medication in her bloodstream to fight an infection. Suppose the initial dosage is 10 mg, and the patient is given an additional maintenance dosage of 4 mg every hour. Assume that the amount of medication in the bloodstream is reduced by 25% every hour.

a. Write a function for the amount of the initial dosage that is in the bloodstream after n hours.

b. Complete the table below to track the amount of medication from the maintenance dosage in the patient's bloodstream for the first five hours.

Hours since initial dose, n	Amount of the medication in the bloodstream from the maintenance dosage at the beginning of each hour
0	0
1	4
2	$4(1 + 0.75)$
3	
4	
5	

c. Write a function that models the total amount of medication in the bloodstream after n hours.

d. Use a calculator to graph the function you wrote in part (c). According to the graph, will there ever be more than 16 mg of the medication present in the patient's bloodstream after each dose is administered?

e. Rewrite this function as the difference of two functions (one a constant function and the other an exponential function), and use that difference to justify why the amount of medication in the patient's bloodstream will not exceed 16 mg after each dose is administered.