Lesson 14: Motion Along a Line—Search Robots Again

Student Outcomes

- Students name several points on a line given by a parametric equation and provide the point-slope equation for a line given by a parametric equation
- Students determine whether lines given parametrically are parallel or perpendicular

Lesson Notes

This is an optional lesson intended to allow students to explore parametric equations and compare them with more familiar linear equations. Parametric equations make both the x- and y-variables in an equation dependent upon a third (independent) variable, usually time. In the search robot scenario revisited here, we use parametric equations to model both the robot's horizontal and vertical motion over a period of time. While this is an optional lesson, it prepares students for the topic of parametric equations in higher-level courses leading toward calculus.

Classwork

Opening Exercises (5 minutes)

We will begin the lesson by having students investigate the image created by a given map as the foundation for parameterizing lines where the values of the x- and y-coordinates depend on time (t).

During this exercise students work independently.

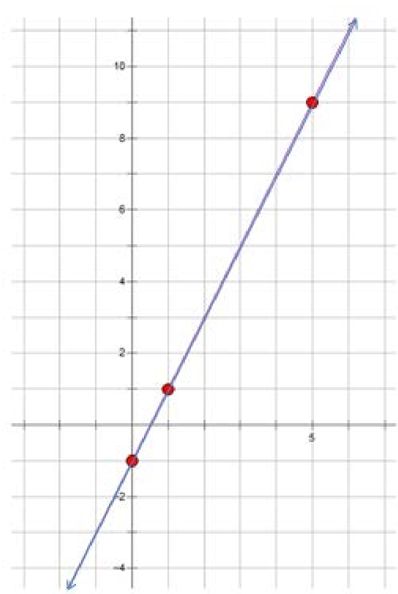
Opening Exercise

a. **If $f(t) = (t, 2t - 1)$, find the values of $f(0)$, $f(1)$, and $f(5)$, and plot them on a coordinate plane.**

$f(0) = (0, -1)$

$f(1) = (1, 1)$

$f(5) = (5, 9)$



b. **What is the image of $f(t)$?**

A line

c. **At what time does the graph of the line pass through the y-axis?**

The line passes through the y-axis when $t = 0$.

d. **When does it pass through the x-axis?**

The line passes through the x-axis when $2t - 1 = 0 \leftrightarrow t = \frac{1}{2}$.

e. Can you write the equation of the line you graphed in slope y-intercept form?

$y = 2x - 1$

f. How does this equation compare with the definition of $f(t)$?

The value of the second coordinate is obtained by taking two times the value of the first coordinate and then subtracting 1.

Example 1 (8 minutes)

MP.4

Example 1

Programmers want to program a robot so that it moves at a uniform speed along a straight line segment connecting two points A and B. If $A(0, -1)$ and $B(1, 1)$, and the robot travels from A to B in $t = 1$minute,

a. Where is the robot at $t = 0$?

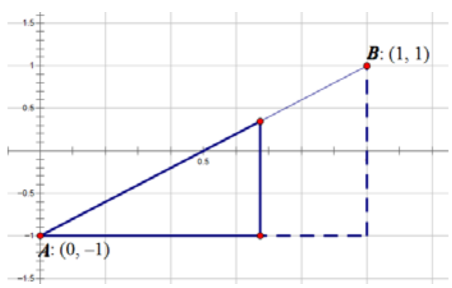
The robot will be at $A(0, -1)$.

b. Where is the robot at $t = 1$?

The robot will be at $B(1, 1)$.

Direct students to choose their own value for t in part (c).

c. Draw a picture that shows where the robot will be at $0 < t < 1$.



Student pictures will vary, but they should all include similar right triangles. Look around the room at the triangles your fellow students sketched.

- What do we know about the robots for those students who selected a location closer to $(1,1)$ than to $(0, -1)$?
  - *These robots have been moving for a longer period of time.*
- Is it possible to find the location of the robot for $t > 1$?
  - *If the robot is moving along a line that passes through points A and B the robot could continue past point B.*

- How far has the robot moved in the horizontal direction after $\frac{1}{2}$ of a minute?
  - *The robot will move half of the entire horizontal distance between point A and point B because it is moving at a uniform rate:* $\frac{1}{2}(1-0)=\frac{1}{2}$.
- How far has the robot moved in the vertical direction after $\frac{1}{2}$ of a minute?
  - *The robot will move half of the entire vertical distance between point A and point B because it is moving at a uniform rate:* $\frac{1}{2}\left(1-(-1)\right)=1$.
- Find the location of the robot for $t=\frac{1}{2}$, after the robot has been moving along segment $\overline{AB}$ for half of a minute.
  - *The robot starts at $(0,-1)$ when $t=0$ and moves $\frac{1}{2}$ unit horizontally and 1 unit vertically. At $t=\frac{1}{2}$, the robot will be at* $\left(0+\frac{1}{2},-1+1\right)$ *or* $\left(\frac{1}{2},0\right)$.

Exercise 1 (4 minutes)

Students extend the work done in Example 1 and discover the equation that will give the coordinate of the robot at any given time t. This includes positions for $t>1$. Students examine the effect speed has on the location of the robot along the path at any given time. Have students work with a partner on Exercise 1.

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Exercise 1

A robot is programmed to move along a straight line path through two points A and B. It travels at a uniform speed that allows it to make the trip from $A(0,-1)$ to $B(1,1)$ in $t=1$ minute. Find the location, P, when

a. $t=\frac{1}{4}$

$\left(0+\frac{1}{4}(1-0),-1+\frac{1}{4}(1-(-1))\right)$

$\left(\frac{1}{4},-\frac{1}{2}\right)$

b. $t=0.7$

$\left(0+0.7(1-0),-1+0.7(1-(-1))\right)$

$(0.7,0.4)$

c. $t=\frac{5}{4}$

$\left(0+\frac{5}{4}(1-0),-1+\frac{5}{4}(1-(-1))\right)$

$\left(\frac{5}{4},\frac{3}{2}\right)$

d. $t=2.2$

$\left(0+2.2(1-0),-1+2.2(1-(-1))\right)$

$(2.2,3.4)$

Scaffolding:
- Struggling learners may benefit from using the diagram they created during Example 1.
- While students work in partners, work with a small group that would benefit from targeted instruction.

Example 2 (6 minutes)

In this example we investigate the impact the speed of the robot has on its location at any given time. We will us this to create a general equation for the location of the robot at any time t.

Example 2

Our robot has been reprogrammed so that it moves along the same straight line path through two points $A(0,-1)$ and $B(1,1)$ at a uniform rate but makes the trip in 0.6 minutes instead of 1 minute.

How does this change the way we calculate the location of the robot at any time, t?

a. **Find the location, P, of the robot from Example 1 if the robot were traveling at a uniform speed that allowed it to make the trip from A to B in $t = 0.6$ minutes. Is the robot's speed greater or less than the robot's speed in Example 1?**

$$P = (a_1+, a_2) + \frac{t}{(0.6)}(b_1 - a_1, b_2 - a_2) \text{ or } P = (a_1, a_2) + \frac{5}{3}t(b_1 - a_1, b_2 - a_2)$$

The robot is moving faster that it was in Example 1 because it travels the same distance from A and B in a shorter period of time.

b. **Find the location, P, of the robot from Example 1 if the robot were traveling at a uniform speed that allowed it to make the trip from A to B in $t = 1.5$ minutes. Is the robot's speed greater or less than the robot's speed in Example 1?**

$$P = (a_1+, a_2) + \frac{t}{1.5}(b_1 - a_1, b_2 - a_2) \text{ or } P = (a_1, a_2) + \frac{2}{3}t(b_1 - a_1, b_2 - a_2)$$

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The robot is moving more slowly that it was in Example 1 because it travels the same distance from A and B in a longer period of time.

Exercise 2 (8 minutes)

This exercise provides students with an opportunity to investigate the effect the straight line path has on the position of the robot.

Exercise 2

Two robots are moving along straight line paths in a rectangular room. Robot 1 starts at point $A(20,10)$ and travels at a constant speed to point $B(120,50)$ in two minutes. Robot 2 starts at point $C(90,10)$ and travels at a constant speed to point $D(60,70)$ in 90 seconds.

a. **Find the location, P, of Robot 1 after it has traveled for t minutes along its path from A to B.**

$$P = \left(20 + \frac{t}{2}(100), 10 + \frac{t}{2}(40)\right)$$

$$P = (20, 10) + \frac{t}{2}(100, 40)$$

b. **Find the location, Q, of Robot 2 after it has traveled for t minutes along its path from A to B.**

$$Q = \left(90 + \frac{t}{1.5}(-30), 10 + \frac{t}{1.5}(60)\right)$$

$$Q = (90, 10) + \frac{t}{1.5}(-30, 60)$$

c. Are the robots traveling at the same speed? If not, which robot's speed is greater?

$AB = \sqrt{(120-20)^2 + (50-10)^2}$
$AB = \sqrt{11600}$
$AB = 20\sqrt{29}$
$AB \approx 107.7$

$CD = \sqrt{(60-90)^2 + (70-10)^2}$
$CD = \sqrt{4500}$
$CD = 30\sqrt{5}$
$CD \approx 67.1$

Robot 1 travels approximately 107.7 *units in two minutes which is about* 53.85 *units per minute.*

Robot 2 travels approximately 67.1 *units in* 1.5 *minutes which is about* 44.7 *units per minute.*

Therefore, Robot 1's speed is greater.

d. Are the straight line paths that the robots are traveling parallel, perpendicular, or neither? Explain your answer.

slope $\overline{AB} = \frac{50-10}{120-20} = \frac{2}{5}$

slope $\overline{CD} = \frac{70-10}{60-90} = -2$

The paths are neither parallel nor perpendicular; the slopes of their straight line paths are not equal nor do they have a product equal to -1.

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Example 3 (4 minutes)

Example 3

A programmer wants to program a robot so that it moves at a constant speed along a straight line segment connecting the point $A(30, 60)$ to the point $B(200, 100)$ over the course of a minute.

At time $t = 0$, the robot is at point A.

At time $t = 1$, the robot is at point B.

a. Where will the robot be at time $t = \frac{1}{2}$?

Because the robot is moving at uniform speed, the robot should travel half the distance between points A *and* B *in half of the time it takes the robot to travel the entire distance.*

$$\left(30 + \frac{1}{2}(200-30), 60 + \frac{1}{2}(100-60)\right) = (115, 80)$$

b. Where will the robot be at time $t = 0.6$?

The robot will be three-fifths of the way along the segment from A *to* B.

$$\left(30 + \frac{3}{5}(200-30), 60 + \frac{3}{5}(100-60)\right) = (132, 84)$$

Discussion (3 minutes)

- In the Opening Exercises, what factors were the key pieces of information you used to determine the location of the robot?
  - *That the robot was moving along a line segment.*
  - *The coordinates of the endpoints of the line segment.*
  - *How long the robot had been moving along the segment.*
  - *The time it took the robot to travel the entire length of the segment.*
- Why did we need to know both the time the robot was traveling and the time it took the robot to travel the entire length of the segment?
  - *The robot will cover a fraction of the length of the segment that is equal to the segment which is equal to the ratio of the time the robot was traveling to the total time it takes the robot to travel the entire length.*
- How would changing the speed the robot was traveling along the segment change the Opening Exercises?
  - *If the robot were traveling faster, the time it takes the robot to travel the entire length of the segment would decrease and the robot would be farther along the segment at each of the given times.*
  - *If the robot were traveling more slowly, the time it takes the robot to travel the entire length of the segment would increase, and the robot would not have traveled as far along the segment at each of the given times.*

Closing (2 minutes)

Ask students to respond to these questions in writing, with a partner, or as a class.

- What role did time play in determining the locations of the robots during today's lesson?
  - *As time increased, the robot's location moved away from its starting position. The x- and y-coordinates both changed.*
- How did we accommodate this parameter when we wrote the expressions that represented the coordinates of the locations?
  - *The coordinates both were dependent on t, so each coordinate was written as a function of the variable t.*

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

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Exit Ticket

Programmers want to program a robot so that it moves along a straight line segment connecting the point $A(35, 80)$ to the point $B(150, 15)$ at a uniform speed over the course of five minutes. Find the location of the robot at the following times:

a. $t = 0$

b. $t = 2$

c. $t = 3.5$

d. $t = 5$

Exit Ticket Sample Solutions

Programmers want to program a robot so that it moves along a straight line segment connecting the point $A(35, 80)$ to the point $B(150, 15)$ at a uniform speed over the course of five minutes. Find the robot's location at the following times:

a. $t = 0$

At $t = 0$, the robot is at point $A(35, 80)$.

b. $t = 2$

At $t = 2$, the robot is at $(35, 80) + \frac{2}{5}(115, -65)$ or $(81, 54)$.

c. $t = 3.5$

At $t = 3.5$, the robot is at $(35, 80) + \frac{3.5}{5}(115, -65)$ or $(115.5, 34.5)$.

d. $t = 5$

At $t = 5$, the robot is at point $B(150, 15)$.

Problem Set Sample Solutions

1. **Find the coordinates of the intersection of the medians of $\triangle ABC$ given $A(2, 4)$, $B(-4, 0)$, and $C(3, -1)$.**

$$\left(\frac{1}{3}(2 + (-4) + 3), \frac{1}{3}(4 + 0 + (-1))\right) = \left(\frac{1}{3}, 1\right)$$

2. **Given a quadrilateral with vertices $A(-1, 3)$, $B(1, 5)$, $C(5, 1)$, and $D(3, -1)$:**

a. **Prove that quadrilateral $ABCD$ is a rectangle.**

$\overline{AB}$ and $\overline{DC}$ have slopes of 1, so they are parallel.

$\overline{BC}$ and $\overline{AD}$ have slopes of -1, so they are parallel.

Since adjacent sides have slopes that are negative reciprocals, they are perpendicular; therefore, each angle is a right angle.

Both pairs of opposite sides are parallel, so the quadrilateral is a parallelogram.

All angles are right angles, so the parallelogram is a rectangle.

b. **Prove that $(2, 2)$ is a point on both diagonals of the quadrilateral.**

Since $ABCD$ is a parallelogram, its diagonals intersect at their midpoints. $(2, 2)$ is the midpoint of both $\overline{AC}$ and $\overline{BD}$, so it is a point on both diagonals.

3. The robot is programed to travel along a line segment at a constant speed. If P represents the robot's position at any given time t in minutes:

$$P = (240, 60) + \frac{t}{10}(100, 100),$$

a. What was the robot's starting position?

Starting position: $(240, 60) + \frac{0}{10}(100, 100)$ *or* $(240, 60)$

b. Where did the robot stop?

Stopping position: $(240, 60) + \frac{10}{10}(100, 100)$ *or* $(340, 160)$

c. How long did it take the robot to complete the entire journey?

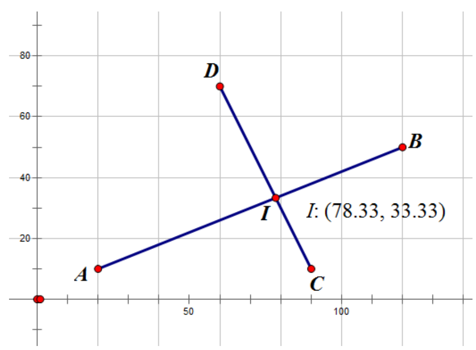
The robot will travel the entire length of the segment in 10 *minutes.*

d. Did the robot pass through the point $(310, 130)$ and, if so, how long into its journey did the robot reach this position?

There are a number of ways to answer part (d). The one most relevant to this lesson might be the following:

Find a time such that $240 + \frac{t}{10}(100) = 310$, *and* $60 + \frac{t}{10}(100) = 130$. *The robot passed through the point* $(310, 130)$ *when* $t = 7$.

4. Two robots are moving along straight line paths in a rectangular room. Robot 1 starts at point $A(20, 10)$ and travels at a constant speed to point $B(120, 50)$ in two minutes. Robot 2 starts at point $C(90, 10)$ and travels at a constant speed to point $D(60, 70)$ in 90 seconds. If the robots begin their journeys at the same time, will the robots collide? Why or why not?



Robot 1's position is modeled by

$$P = (20, 10) + \frac{t}{2}(100, 40).$$

Robot 2's position is modeled by

$$Q = (90, 10) + \frac{t}{1.5}(-30, 60).$$

$$(20, 10) + \frac{t}{2}(100, 40) = \left(78\frac{1}{3}, 33\frac{1}{3}\right)$$

Robot 1 passes through the point $\left(78\frac{1}{3}, 33\frac{1}{3}\right)$ *at* $t = 1\frac{1}{6}$.

$$(90, 10) + \frac{t}{1.5}(-30, 60) = \left(78\frac{1}{3}, 33\frac{1}{3}\right)$$

Robot 2 passes through the point $\left(78\frac{1}{3}, 33\frac{1}{3}\right)$ *at* $t = \frac{7}{12}$.

The paths of the two robots do intersect, but the robots reach this point $\left(78\frac{1}{3}, 33\frac{1}{3}\right)$ *at different times, so the robots will not collide.*