Lesson 13: Analytic Proofs of Theorems Previously Proved by Synthetic Means

Classwork

Opening Exercise

Let $A(30,40)$, $B(60,50)$, and $C(75,120)$ be vertices of a triangle.

a. Find the coordinates of the midpoint M of $\overline{AB}$ and the point G_1 that is the point one-third of the way along $\overline{MC}$, closer to M than to C.

b. Find the coordinates of the midpoint N of $\overline{BC}$ and the point G_2 that is the point one-third of the way along $\overline{NA}$, closer to N than to A.

c. Find the coordinates of the midpoint R of $\overline{CA}$ and the point G_3 that is the point one-third of the way along $\overline{RB}$, closer to R than to B.

Exercise 1

a. Given triangle ABC with vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$, find the coordinates of the point of concurrency.

b. Let $A(-23, 12)$, $B(13, 36)$, and $C(23, -1)$ be vertices of a triangle. Where will the medians of this triangle intersect?

Exercise 2

Prove that the diagonal of a parallelogram bisect each other.

Problem Set

1. Point M is the midpoint of segment $\overline{AC}$. Find the coordinates of M:
   a. $A(2,3), C(6,10)$
   b. $A(-7,5), C(4,-9)$

2. $M(-2,10)$ is the midpoint of segment $\overline{AB}$. If A has coordinates $(4,-5)$, what are the coordinates of B?

3. Line A is the perpendicular bisector of segment $\overline{BC}$ with $B(-2,-1)$ and $C(4,1)$.
   a. What is the midpoint of $\overline{BC}$?
   b. What is the slope of $\overline{BC}$?
   c. What is the slope of line A? (Remember, it is perpendicular to $\overline{BC}$.)
   d. Write the equation of line A, the perpendicular bisector of $\overline{BC}$.

4. Find the coordinates of the intersection of the medians of $\triangle ABC$ given $A(-5,3)$, $B(6,-4)$, and $C(10,10)$.

5. Use coordinates to prove that the diagonals of a parallelogram meet at the intersection of the segments that connect the midpoints of its opposite sides.

6. Given a quadrilateral with vertices $E(0,5)$, $F(6,5)$, $G(4,0)$, and $H(-2,0)$:
   a. Prove quadrilateral $EFGH$ is a parallelogram.
   b. Prove $(2,2.5)$ is a point on both diagonals of the quadrilateral.

7. Prove quadrilateral $WXYZ$ with vertices $W(1,3)$, $X(4,8)$, $Y(10,11)$, and $Z(4,1)$ is a trapezoid.

8. Given quadrilateral $JKLM$ with vertices $J(-4,2)$, $K(1,5)$, $L(4,0)$, and $M(-1,-3)$:
   a. Is it a trapezoid? Explain.
   b. Is it a parallelogram? Explain.
   c. Is it a rectangle? Explain.
   d. Is it a rhombus? Explain.
   e. Is it a square? Explain.
   f. Name a point on the diagonal of $JKLM$. Explain how you know.