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Lesson 7: Equations for Lines Using Normal Segments

Student Outcomes

* Students state the relationship between previously used formats for equations for lines and the new format

12, recognizing the segments from to as a normal and as a slope.

* Students solve problems that are dependent upon making such interpretations.

Lesson Notes

This lesson focuses on MP.4 because students work extensively to model robot behavior using coordinates.

Classwork

**Opening Exercise (5 minutes)**

This exercise can be modeled by the teacher with the whole class, given to groups to present solutions to the class, or used as a supplement to the lesson.

Opening Exercise

**The equations given are in standard form. Put each equation in slope-intercept form. State the slope and the   
-intercept.**

*Scaffolding:*

Provide visuals to reinforce standard and slope-intercept forms*:*

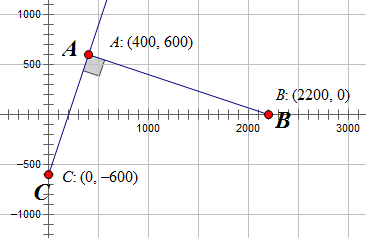
* Standard form is where , , and are integers.
* Slope-intercept form is where is the slope, and is the -intercept.

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| --- | --- | --- |
|  |  |  |
|  |  |  |
| slope | slope | slope |
| -intercept | -intercept | -intercept |

**Discussion (20 minutes)**

*Scaffolding:*

Plotting points will help students visualize the problems and understand the translations required.

Let’s revisit the robot from Lesson 4. Recall that it is moving along the line   
. At the point it detected the loudest “ping,” and the programmers had the robot change direction at that point and move along a linear path that was perpendicular to his original path.

At the end of Lesson 6, we learned that the beacon happens to lie on the -axis and is located at the point .

* How did we determine the location of the beacon?
  + *We had to find the point on the -axis so that and were perpendicular. We knew and and that the -coordinate of point was since point was on the  
    -axis. Using , we eventually determined that .*
* Let’s push this idea a little further. Suppose is any point on the line containing segment . What can you say about and ?
  + *They are also perpendicular.*
* Using, ), and , translate the points so that is at the origin. What are the new coordinates?
  + *If we translate all points in the figure using the translation vector that takes to the origin , then becomes , and becomes .*
* If the condition for perpendicularity is , how could we write an equation involving and ?
  + *The condition that and are perpendicular becomes:*
* This is the equation in standard form: . Look at the values of and and at the work above. Do you see a relationship between and and the work above?
  + *The coordinates of (the translation of ) are , which are the same values as and .*
* Let’s say that in a more specific way. I will state the relationship, and you repeat it and explain it to your partner.
  + *is the -coordinate (abscissa) of the image of the point of the perpendicular segment that does not lie on the line when the point of perpendicularity is at the origin.*
  + *is the -coordinate (ordinate) of the image of the point of the perpendicular segment that does not lie on the line when the point of perpendicularity is at the origin.*
* Now, let’s put the equation in slope-intercept form.
  + *, which is the equation of .*

**MP.7**

* What does the equation you wrote represent?
  + *will be perpendicular to as long as point lies on the line containing , which is given by the equation*

In the next part of this lesson, we generalize what we just discovered. This can be done in several ways.

1. Present the question and explain that we are trying to make this process work for any points with coordinates   
   ,, and . Allow time for students to think and talk to their neighbors for a few minutes; then, show the diagrams and give them more time to talk. Finally, pull everyone together and discuss each step as a class.
2. Assign some groups the task with no leading questions, and let them work independently while other groups are getting different levels of help, some even being directly instructed by the teacher.

* How can we generalize this finding?
* Given point which lies on line , point not on line , and perpendicular to line , then any point on line will satisfy the relationship . Draw the picture described.
* Translate the points. Which point should be on the origin? What is the translation used? What are the coordinates of the translated points?
  + *is the common point and should be translated to the origin. The translation is , or left a and down . The translated points are , , and .*
* If the segments are perpendicular, write the equation that must hold true.
* If and , write this equation substituting in and . Which line have we written the equation of?

*Scaffolding:*

* Have students leave the equations in standard form.
* Provide these steps:

Simplify the parentheses without variables.

Separate the variables—put on one side and on the other.

Distribute the coefficients.

Bring the constant on the right to the left.

Divide by the coefficient of y.

Simplify if necessary.

* + *We end up with the equation of the line that passes through point that is perpendicular to :*
* What do and represent graphically?
  + *is the abscissa, and is the ordinate of the image of .*
* We call segment a normal segment to line because it has one endpoint on the line and is perpendicular to the line.
  + *Explain to your neighbor what a normal segment is, and write your own definition.*

Definition: A line segment with one endpoint on a line and perpendicular to the line is called a **normal** **segment to the line**.

**Example 1 (5 minutes)**

Example 1

Given and :

* 1. Find an equation for the line through and perpendicular to .

,,and

* 1. Find an equation for the line through and perpendicular to .

,, and

Exercises 1–2 (8 minutes)

Have students plot points to aid in solving problems.

1. Given and:
   1. Write an equation for the line through and perpendicular to .

,,

* 1. Write an equation for the line through and perpendicular to .

,,

1. Givenand:
   1. Write an equation for the line through and perpendicular to .

,,

* 1. Write an equation for the line through and perpendicular to .

Closing (2 minutes)

Describe the characteristics of a normal segment.

A line segment with one endpoint on a line and perpendicular to the line is called a normal segment to the line.

Every equation of a line through a given point has the form . Explain how the values of and are obtained.

A is the value of the abscissa of the image of the endpoint of the normal segment that does not lie on the line after the figure has been translated so that the image of the endpoint that does lie on the line, the point of perpendicularity, is at the origin.

B is the value of the ordinate of the image of the endpoint of the normal segment that does not lie on the line after the figure has been translated so that the image of the endpoint that does lie on the line, the point of perpendicularity, is at the origin.

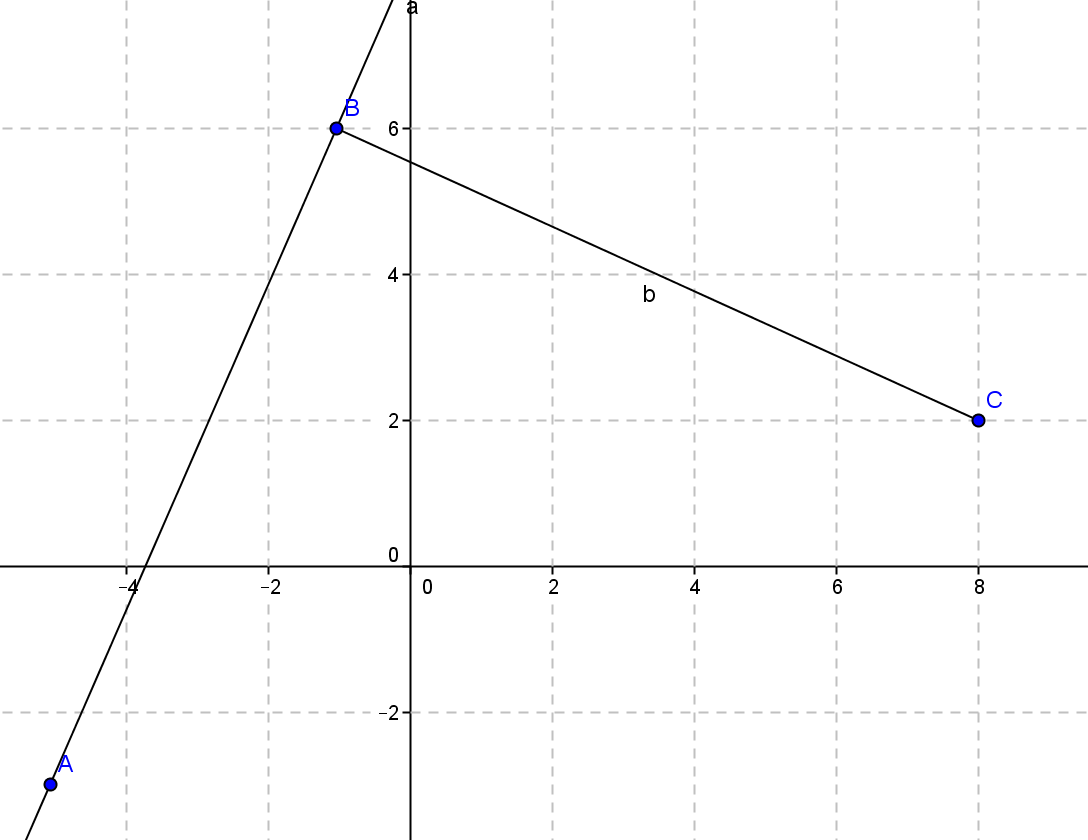
Exit Ticket (5 minutes)

Name Date

Lesson 7: Equations for Lines Using Normal Segments

Exit Ticket

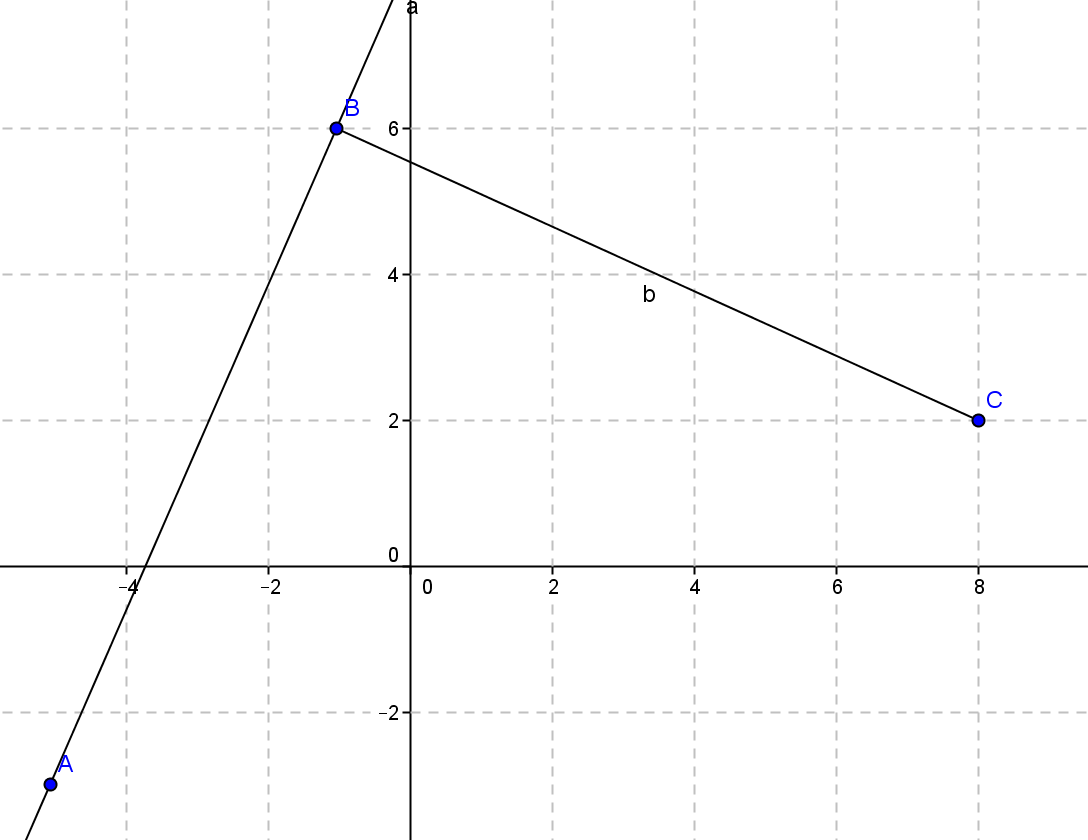
Given ,, and



1. What are the coordinates of the translated points if moves to the origin?
2. Write the condition for perpendicularity equation.
3. Write the equation for the normal line.

Exit Ticket Sample Solutions

Given and:



* 1. What are the coordinates of the translated points if moves to the origin?

,,

* 1. Write the condition for perpendicularity equation.
  2. Write the equation for the normal line in slope-intercept form.

Problem Set Sample Solutions

1. Given pointsand:
   1. Write the equation of the line through and perpendicular to .
   2. Write the equation of the line through and perpendicular to .
2. Given points and:
   1. Write the equation of the line through and perpendicular to .
   2. Write the equation of the line through and perpendicular to .
3. The equation of a line is given by the equation
   1. What are the coordinates of the image of the endpoint of the normal segment that does not lie on the line? Explain your answer.

because is the original formula, and are the coordinates of the image of the endpoint of the normal segment not on the line.

* 1. What translation occurred to move the point of perpendicularity to the origin?

, or left up

* 1. What were the coordinates of the original point of perpendicularity? Explain your answer.

because the translation of was required to move the point of perpendicularity to the origin.

* 1. What were the endpoints of the original normal segment?

, and .

The endpoints of a normal segment to the given line are and.

1. A coach is laying out lanes for a race. The lands are perpendicular to a segment of the track such that one endpoint of the segment is and the other is . What are the equations of the lines through the endpoints?