

Lesson 6: Segments That Meet at Right Angles

Classwork

Opening Exercise

Carlos thinks that the segment having endpoints $A(0,0)$ and $B(6,0)$ is perpendicular to the segment with endpoints $A(0,0)$ and $C(-2,0)$. Do you agree? Why or why not?

Working with a partner, given $A(0,0)$ and $B(3,-2)$, find the coordinates of a point C so that $\overline{AC} \perp \overline{AB}$.

Example 1

Given points $A(2,2)$, $B(10,16)$, $C(-3,1)$, and $D(4,-3)$, are segments \overline{AB} and \overline{CD} perpendicular? Are the lines containing the segments perpendicular? Explain.

Exercises 1–4

1. Given $A(a_1, a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$, and $D(d_1, d_2)$, find a general formula in terms of $a_1, a_2, b_1, b_2, c_1, c_2, d_1$, and d_2 that will let us determine whether segments \overline{AB} and \overline{CD} are perpendicular.

2. Recall the Opening Exercise of Lesson 4 in which a robot is traveling along a linear path given by the equation $y = 3x - 600$. The robot hears a ping from a homing beacon when it reaches the point $B(400, 600)$ and turns to travel along a linear path given by the equation $y - 600 = -\frac{1}{3}(x - 400)$. If the homing beacon lies on the x -axis, what is its exact location? (Use your own graph paper to visualize the scenario.)
 - a. If point A is the y -intercept of the original equation, what are the coordinates of point A ?

 - b. What are the endpoints of the original segment of motion?

 - c. If the beacon lies on the x -axis, what is the y -value of this point, C ?

 - d. Translate point B to the origin. What are the coordinates of A' , B' , and C' ?

 - e. Use the formula derived in this lesson to determine the coordinates of point C .

3. A triangle in the coordinate plane has vertices $A(0, 10)$, $B(-8, 8)$, and $C(-3, 5)$. Is it a right triangle? If so, at which vertex is the right angle? (Hint: Plot the points and draw the triangle on a coordinate plane to help you determine which vertex is the best candidate for the right angle.)
4. $A(-7, 1)$, $B(-1, 3)$, $C(5, -5)$, and $D(-5, -5)$ are vertices of a quadrilateral. If \overline{AC} bisects \overline{BD} , but \overline{BD} does not bisect \overline{AC} , determine whether $ABCD$ is a kite.

Problem Set

1. Are the segments through the origin and the points listed perpendicular? Explain.
 - a. $A(9, 10)$, $B(10, 9)$
 - b. $C(9, 6)$, $D(4, -6)$
2. Given $M(5, 2)$, $N(1, -4)$, and L listed below, are segments \overline{LM} and \overline{MN} perpendicular? Translate M to the origin, write the coordinates of the images of the points, then explain without using slope.
 - a. $L(-1, 6)$
 - b. $L(11, -2)$
 - c. $L(9, 8)$
3. Is triangle PQR , where $P(-7, 3)$, $Q(-4, 7)$, and $R(1, -3)$, a right triangle? If so, which angle is the right angle? Justify your answer.
4. A quadrilateral has vertices $(2 + \sqrt{2}, -1)$, $(8 + \sqrt{2}, 3)$, $(6 + \sqrt{2}, 6)$, and $(\sqrt{2}, 2)$. Prove that the quadrilateral is a rectangle.
5. Given points $G(-4, 1)$, $H(3, 2)$, and $I(-2, -3)$, find the x -coordinate of point J with y -coordinate 4 so that the lines \overleftrightarrow{GH} and \overleftrightarrow{IJ} are perpendicular.
6. A robot begins at position $(-80, 45)$ and moves on a path to $(100, -60)$. It turns 90° counterclockwise.
 - a. What point with y -coordinate 120 is on this path?
 - b. Write an equation of the line after the turn.
 - c. If it stops to charge on the x -axis, what is the location of the charger?
7. Determine the missing vertex of a right triangle with vertices $(6, 2)$ and $(5, 5)$ if the third vertex is on the y -axis. Verify your answer by graphing.
8. Determine the missing vertex for a rectangle with vertices $(3, -2)$, $(5, 2)$, and $(-1, 5)$, and verify by graphing. Then, answer the questions that follow.
 - a. What is the length of the diagonal?
 - b. What is a point on both diagonals in the interior of the figure?
9. A right triangle has vertices $(1, 3)$ and $(6, -1)$ and a third vertex located in Quadrant IV.
 - a. Determine the coordinates of the missing vertex.
 - b. Reflect the triangle across the y -axis. What are the new vertices?
 - c. If the original triangle is rotated 90° counterclockwise about the vertex $(6, -1)$, what are the coordinates of the other vertices?
 - d. Now rotate the original triangle 90° clockwise about $(6, -1)$. What are the coordinates of the other vertices?
 - e. What do you notice about both sets of vertices? Explain what you observe.