Lesson 1: Searching a Region in the Plane

Student Outcomes

- Given a physical situation (e.g., a room of a certain shape and dimensions, with objects at certain positions and a robot moving across the room), students impose a coordinate system and describe the given in terms of polygonal regions, line segments, and points **in the coordinate system**.

Lesson Notes

This lesson will require students to remember concepts such as right triangle trigonometry, graphing on the coordinate plane, and the Pythagorean theorem, and formulas such as the distance formula ($d = \sqrt{a^2 + b^2}$), and the rate formula ($rate \times time = distance$). Some students may need to be reminded how to write the equation of a line. The task at first seems complicated but can be broken down easily if students use graph paper and scale the graph down to show very specific points given.

MP.4 In this lesson, students transition from verbal, graphical, and algebraic thinking to modeling robot motion in a straight line and using lines of motion and previously learned topics (distance, proportion) to determine the location of impact in a warehouse.

Classwork

Opening (8 minutes)

What type of math (geometry) do you think is involved in programming a robot to vacuum an empty room?

Have students brainstorm and write the mathematical terms they suggest on the board. Do not comment on the ideas; simply write the terms given.

Students will have varying levels of familiarity with robots; show some or all of this video to generate interest and other ideas about the mathematics involved in programming: http://www.youtube.com/watch?v=gvQKGev56qU. After showing the video, ask the initial question again. Continue writing terms on the board, but this time, have students explain how the mathematics given would be used.

If students are hesitant to participate, provide scaffolds, such as the following sentence starters:

- To determine how far the robot moves, I could …
- To identify where the robot starts and ends, I could …
- To explain the path of the robot, I could …
- To find the speed of the robot, I could …

Ask leading questions to elicit the responses below if they are not mentioned by students.

  - *Distance, angles, slope, lines, equations of lines, graphing on the coordinate plane, scale, rate, time, triangles, and right triangle trigonometry.*

Exploratory Challenge (27 minutes): Programming a Robot

This example illustrates the types of problems we will be exploring and studying in this module—using mathematics to describe regions in the plane (rooms) and using geometry to fully understand linear motion. This is an exploration activity. Students will work in heterogeneous groups of two or three while the teacher circulates around to answer questions, and then brings the class back together to discuss different strategies for solving and answering each part of the problem.

Present the example and allow five minutes for students to re-read the problem and discuss strategies with their group for solving the problem. Encourage them to brainstorm and do a 30 second quick-write about initial strategies. Circulate and listen to discussions. Bring the class back together and have students share ideas. Use the discussions to analyze where groups are in the problem-solving process. Students then return to their groups and begin working.

Give students graph paper to start and have them properly scale the axes and plot the known points. Use the exercises as scaffolds to guide students as they work. Groups will need varying levels of scaffolding. Some groups may answer all of the questions and some just a few. Continue walking around the room and answering questions or asking guiding questions where appropriate such as, "If a robot could travel 10 feet in 1 second, what is its rate of speed?" or "If a robot could travel 400 feet in 20 seconds, what is its rate of speed?" Let students really think through the solutions. As you monitor student progress, some strategies that could be used to help groups or the entire class are the following: pause the groups after every five minutes of work time and ask for group updates on what is working and what is not working; call team leaders to huddle and give hints; station yourself near a struggling group to guide them; and pair groups to share ideas and generate new ideas. Remind students that the robot's motion is constant.

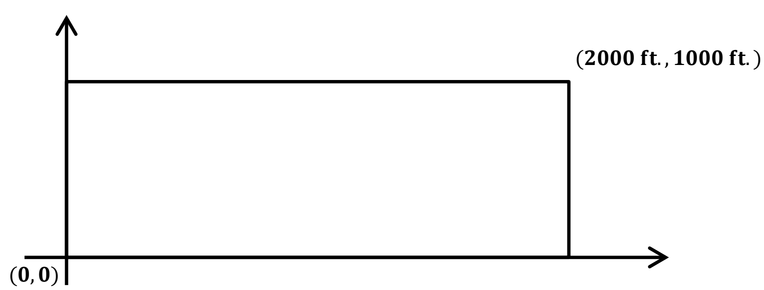
Scaffolding:

- If students are struggling, refer to Grade 5, Module 6, Lessons 2, 3, and 4. For a fun review activity on graphing, set up your room as a coordinate plane and have students plot themselves and objects in the room.
- Provide smaller sized, labeled graph paper.
- As an extension, have students determine paths of motion to move from given points to other points using translations.

MP.4

Exploratory Challenge

Students in a robotics class must program a robot to move about an empty rectangular warehouse. The program specifies location at a given time, t, seconds. The room is twice as long as it is wide. Locations are represented as points in a coordinate plane with the southwest corner of the room deemed the origin, $(0, 0)$, and the northeast corner deemed the point $(2000 \text{ ft.}, 1000 \text{ ft.})$, as shown in the diagram below.



The first program written has the robot moving at a constant speed in a straight line. At time $t = 1$ second, the robot is at position $(30, 45)$, and at $t = 3$ seconds, it is at position $(50, 75)$. Complete the exercises and answer the questions below to program the robot's motion.

a. **Where is the location of impact?**

It is at position $\left(666\frac{2}{3}, 1000\right)$.

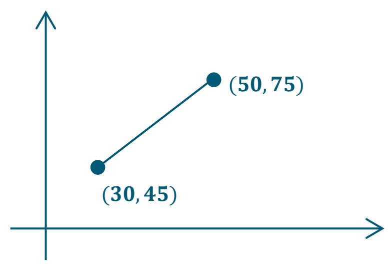
b. **At what speed will the robot hit the wall?**

It will hit at approximately 18 *ft/sec.*

c. **At what time will the robot hit the wall?**

It will hit at about $t = 64.8$ *seconds.*

Exercises 1–8



1. **Plot the points on a coordinate plane.**

2. **Draw the line connecting the segments.**

3. **How much did the x-coordinate change in 2 seconds?**

   It changed 20 *ft.*

4. **How much did the y-coordinate change in 2 seconds.**

   It changed 30 *ft.*

5. **What is the ratio of change in y to change in x?**

   The ratio of change is $\frac{3}{2}$.

6. **What is the equation of the line of motion?**

   The equation is $y = \frac{3}{2}x$.

7. **What theorem could be used to find the distance between the points?**

   The Pythagorean theorem could be used.

8. **How far did the robot travel in 2 seconds?**

   It traveled ≈ 36.06 *ft.*

Students intuitively know that if they let the robot continue on with this motion, it will hit the wall. The questions below will give them information so that they can program the robot. This discussion should begin after groups have completed the exercises and as they are starting to begin answering the example questions. The questions can be answered in a class discussion or in groups, depending on the needs of your students. Have students predict the location of impact and then verify their answer algebraically. Use the questions listed as scaffolds to the discussion.

- Predict the location of impact based on the diagram and then verify your answer algebraically.
- Which wall will it hit? Explain.
  - *It will hit the top wall because if the line is extended, it intersects the top wall first.*
- Where will it hit? Explain.
  - *Answers will vary. Some students may give estimates of* $(600, 1000)$*, some may say* x *is between* 600 *and* 700*, but* y *is always* 1000*, and some may do the calculation immediately.*
- Where will the robot be at $t = 23$ seconds? Explain.
  - *Rate times time gives us distance, so* $18\ \frac{ft}{sec} \times 23\ seconds = 414\ feet$
- What can we use to find the location of impact? Explain.
  - *We can use the equation of the line of motion with* $y = 1000$*.*
- What is needed to calculate speed? Explain.
  - *Distance and time are needed. Speed is equal to the quotient of distance and time.*
- What are the units of speed in this problem?
  - *The units of speed are feet/seconds.*
- How far did the robot travel between the two points given? How did you calculate that?
  - *It traveled* ≈ 36.06 *ft. Using the distance formula, you have*
    $$d = \sqrt{(50-30)^2 + (75-45)^2}.$$
- How long did it take the robot to move this distance?
  - $36.06\ ft \div 18\ \frac{ft}{sec} \approx 2\ seconds.$
- If we know distance and time, how can we find rate?
  - *Use* $distance \div time$*.*
- What do we need to calculate time?
  - *We need rate and distance.*
- Where did the robot start its motion? At what time did it start?
  - *Using the graph, students should see that the* x*-coordinate is changing* 10 *feet each second and the* y*-coordinate is changing* 15 *feet each second. If the robot is at* $(30,45)$ *after* 1 *second, then it started at* $(20, 30)$ *at time* $t = 0$*.*
- What is the distance from $(20, 30)$ to the wall?
  - *Students must use the distance formula to find the distance between* $(20, 30)$ *and* $\left(666\frac{2}{3}, 1000\right) \approx 1111.71$ *ft.*
- What is the constant rate?
  - *The constant rate is* $18\ \frac{ft}{sec}$*.*
- Knowing distance and rate, how can you find time?
  - *Use* $distance \div time$*.*

If any group finishes early, have the students program a robot to travel around the classroom and pick up certain objects. Have students measure the classroom and locate objects on a coordinate plane, then program their own robot. They should record their information on a poster to share with the class. This activity could be done in groups of four, pairing the groups in the order that they finish the above example.

Closing (5 minutes)

Ask students to respond to these questions in writing, to a partner, or as a class.

- When programming a robot, what needs to be known to calculate the speed? What theorem helps you find necessary information?
  - *Since the units of speed are ft/sec in this example, we would need to find distance in feet and know the time the robot needed to travel that distance to calculate speed. The Pythagorean theorem and distance formula allow us to calculate distance. Students may also realize that speed is the slope of this line of motion.*
- What are some methods that can be used to determine where the robot will hit the wall? Explain.
  - *If the equation of motion is known or can be calculated, substitute in the boundary coordinate of the side of the room the robot will hit. For our example, it hit the top wall that had a y-coordinate of 1000. If it had hit the right wall, we would have substituted $x = 2000$.*

Exit Ticket (5 minutes)

Name ______________________________ Date ________________

Lesson 1: Searching a Region in the Plane

Exit Ticket

You are moving the robot to your classroom, which measures 30 feet by 40 feet.

1. Draw the classroom set up on a coordinate plane using $(30, 40)$ as the northeast vertex.

2. The robot was initially placed at position $(6, 9)$, and at $t = 2$ seconds, its position is $(10, 15)$.
   a. How far did the robot travel in 2 seconds?

   b. What is the speed of the robot?

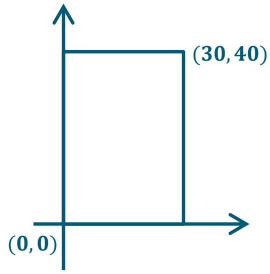
Exit Ticket Sample Solutions

Scaffolding:

- Have the robot start at $(0, 0)$ and move to $(2, 2)$ in a 10 ft. $\times$ 10 ft. room.
- For students with spatial or eye-hand coordination difficulties, just assess with Problem 2, and provide them the completed grid for Problem 1.

You are moving the robot to your classroom, which measures 30 feet by 40 feet.

1. **Draw the classroom set up on the coordinate plane using $(30, 40)$ as the northeast vertex.**



2. **The robot was initially placed at position $(6, 9)$, and at $t = 2$ seconds, its position is $(10, 15)$.**

   a. **How far did the robot travel in 2 seconds?**

   It traveled approximately 7.2 ft.

   b. **What is the speed of the robot?**

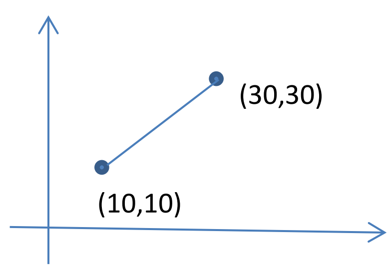
   It traveled approximately 3.6 ft/sec.

Problem Set Sample Solutions

Encourage students to use graph paper to help them get started on these problems.

1. **The robot in the video is moving around an empty 100 ft. by 100 ft. storage room at a constant speed. If the robot crosses $(10, 10)$ at 1 second and $(30, 30)$ at 6 seconds:**

   a. **Plot the points and draw the segment connecting the points.**



   b. **What was the change in the x-coordinate?**

   It changed 20 ft.

   c. **What was the change in the y-coordinate?**

   It changed 20 ft.

   d. **What is the ratio of the change in y to the change in x?**

   The ratio of change is $1:1$.

   e. **How far did the robot travel between the two points?**

   It traveled ≈ 28.28 ft.

f. **What was the speed of the robot?**

The speed was ≈ 5.66 *ft./sec.*

g. **Where did the robot start?**

The robot started at position $(6, 6)$.

2. **Your mother received a robot vacuum cleaner as a gift and wants you to help her program it to clean a vacant 30 ft. by 30 ft.room. If the vacuum is at position $(12, 9)$ at time $t = 2$ seconds and at position $(24, 18)$ at $t = 5$ seconds, answer the following:**

a. **How far did the robot travel over 3 seconds?**

It traveled 15 *ft.*

b. **What is the speed?**

The speed is 5 *ft/sec.*

c. **What is the ratio of the change in the x-coordinate to the change in the y-coordinate?**

The ratio of change is $\frac{4}{3}$, *or* $y = \frac{3}{4}x$.

d. **Where did the robot start?**

It started at position $(4, 3)$.

e. **Where will the robot be at $t = 3$ seconds? Explain how you know.**

It will be at position $(16, 12)$. *I know that each second the* x*-value changed* 4 *units and the* y*-value changed* 3 *units. The* x*-value started at* 4*; therefore, in* 3 *seconds, it increased* 3×4 *units, which means the* x*-value at* 3 *seconds is* $4 + 12$, *or* 16. *The* y*-value started at* 3 *and increased* 3×3 *units, giving a* y*-value of* $3 + 9$, *or* 12.

f. **What is the location of impact?**

The location of impact is $\left(30, 22\frac{1}{2}\right)$.

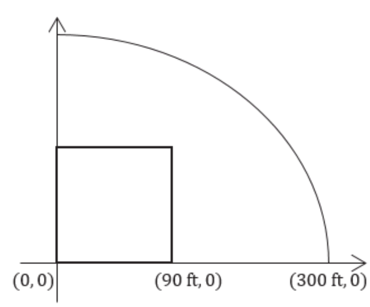
g. **At what time will the robot hit the wall?**

It will hit the wall at about $t = 6.5$ *seconds.*

3. **A baseball player hits a ball at home plate at position $(0, 0)$. It travels at a constant speed across first base at position $(90, 0)$ in 2 seconds.**

a. **What was the speed of the ball?**

The speed was 45 *ft/sec.*



b. When will it cross the fence at position $(300, 0)$? Explain how you know.

It will cross the fence in ≈ 6.67 seconds. I know because the distance is 300 feet at a constant speed of 45 ft/sec. $300 \div 45 \approx 6.67$ seconds.

4. The tennis team has a robot that picks up tennis balls. The tennis court is 36 feet wide and 78 feet long. The robot starts at position $(8, 10)$ and is at position $(16, 20)$ at $t = 4$ seconds. When will it pick up the ball located at position $(28, 35)$?

It will pick up that ball in approximately 6 seconds.