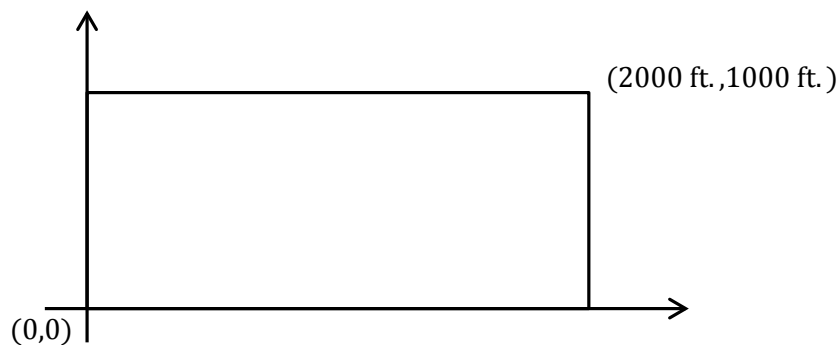


Lesson 1: Searching a Region in the Plane

Classwork

Exploratory Challenge

Students in a robotics class must program a robot to move about an empty rectangular warehouse. The program specifies location at a given time, t , seconds. The room is twice as long as it is wide. Locations are represented as points in a coordinate plane with the southwest corner of the room deemed the origin, $(0,0)$, and the northeast corner deemed the point $(2000 \text{ ft.}, 1000 \text{ ft.})$, as shown in the diagram below.



The first program written has the robot moving at a constant speed in a straight line. At time $t = 1$ second, the robot is at position $(30, 45)$, and at $t = 3$ seconds, it is at position $(50, 75)$. Complete the exercises and answer the questions below to program the robot's motion.

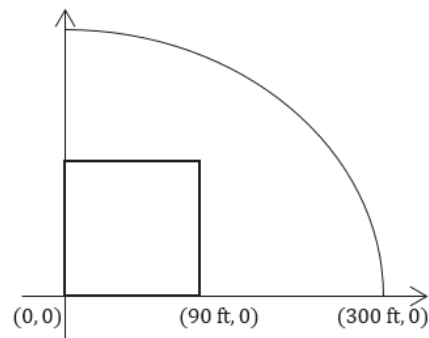
- Where is the location of impact?
- At what speed will the robot hit the wall?
- At what time will the robot hit the wall?

Exercises 1–8

1. Plot the points on a coordinate plane.
2. Draw the line connecting the segments.
3. How much did the x -coordinate change in 2 seconds?
4. How much did the y -coordinate change in 2 seconds?
5. What is the ratio of change in y to change in x ?
6. What is the equation of the line of motion?
7. What theorem could be used to find the distance between the points?
8. How far did the robot travel in 2 seconds?

Problem Set

1. The robot in the video is moving around an empty 100 ft. by 100 ft. storage room at a constant speed. If the robot crosses $(10, 10)$ at 1 second and $(30, 30)$ at 6 seconds:
 - a. Plot the points and draw the segment connecting the points.
 - b. What was the change in the x -coordinate?
 - c. What was the change in the y -coordinate?
 - d. What is the ratio of the change in y to the change in x ?
 - e. How far did the robot travel between the two points?
 - f. What was the speed of the robot?
 - g. Where did the robot start?
2. Your mother received a robot vacuum cleaner as a gift and wants you to help her program it to clean a vacant 30 ft. by 30 ft. room. If the vacuum is at position $(12, 9)$ at time $t = 2$ seconds and at position $(24, 18)$ at $t = 5$ seconds, answer the following:
 - a. How far did the robot travel over 3 seconds?
 - b. What is the speed?
 - c. What is the ratio of the change in the x -coordinate to the change in the y -coordinate?
 - d. Where did the robot start?
 - e. Where will the robot be at $t = 3$ seconds? Explain how you know.
 - f. What is the location of impact?
 - g. At what time will the robot hit the wall?
3. A baseball player hits a ball at home plate at position $(0, 0)$. It travels at a constant speed across first base at position $(90, 0)$ in 2 seconds.
 - a. What was the speed of the ball?
 - b. When will it cross the fence at position $(300, 0)$? Explain how you know.



4. The tennis team has a robot that picks up tennis balls. The tennis court is 36 feet wide and 78 feet long. The robot starts at position $(8, 10)$ and is at position $(16, 20)$ at $t = 4$ seconds. When will it pick up the ball located at position $(28, 35)$?

Lesson 2: Finding Systems of Inequalities That Describe Triangular and Rectangular Regions

Classwork

Opening Exercise

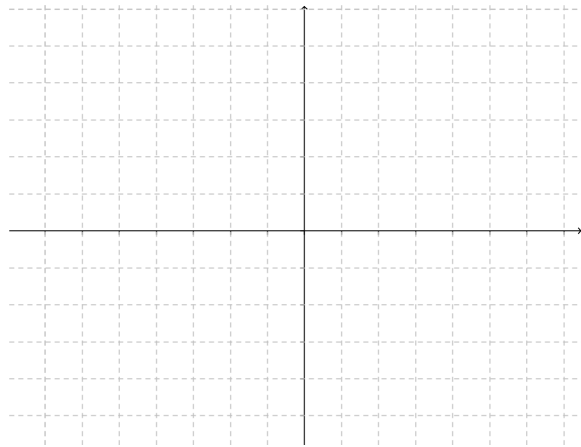
Graph each system of inequalities.

1.
$$\begin{cases} y \geq 1 \\ x \leq 5 \end{cases}$$

a. Is $(1, 2)$ a solution? Explain.

b. Is $(1, 1)$ a solution? Explain.

c. The region is the intersection of how many half-planes? Explain how you know.

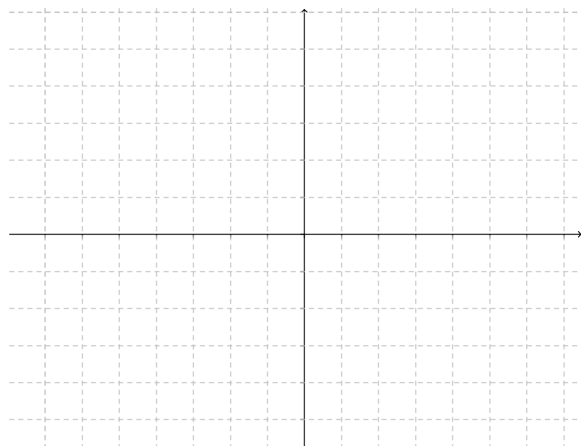


2.
$$\begin{cases} y < 2x + 1 \\ y \geq -3x - 2 \end{cases}$$

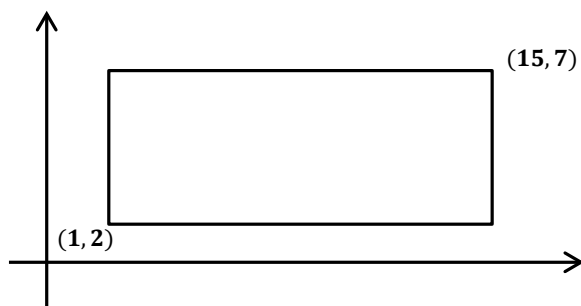
a. Is $(-2, 4)$ in the solution set?

b. Is $(1, 3)$ in the solution set?

c. The region is the intersection of how many half-planes? Explain how you know.

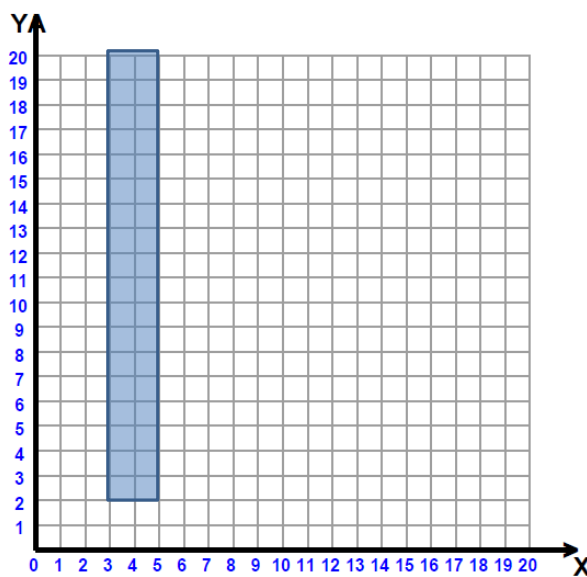


Example 1



Exercises 1–3

1. Given the region shown to the right:
 - a. Name three points in the region.
 - b. Name three points on the boundary.
 - c. Explain in words the points in the region.
 - d. Write the inequality describing the x -values.
 - e. Write the inequality describing the y -values.



f. Write this as a system of equations.

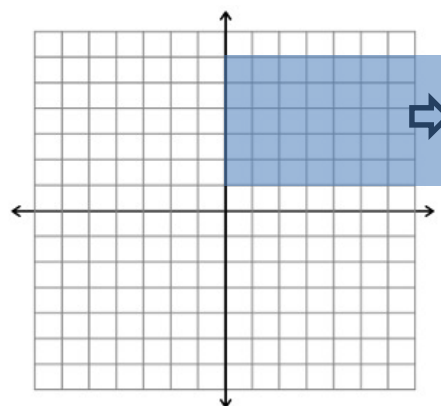
g. Will the lines $x = 4$ and $y = 1$ pass through the region? Draw them.

2. Given the region that continues unbound to the right as shown to the right:

a. Name three points in the region.

b. Describe in words the points in the region.

c. Write the system of inequalities that describe the region.



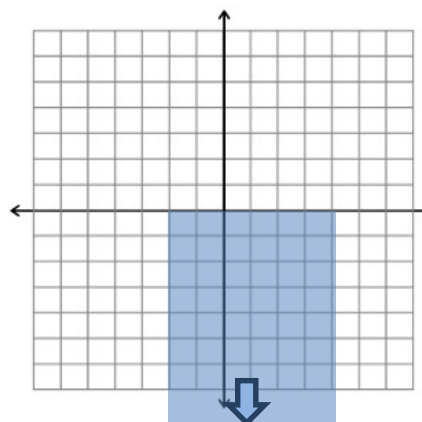
d. Name a horizontal line that passes through the region.

3. Given the region that continues down without bound as shown to the right:

a. Describe the region in words.

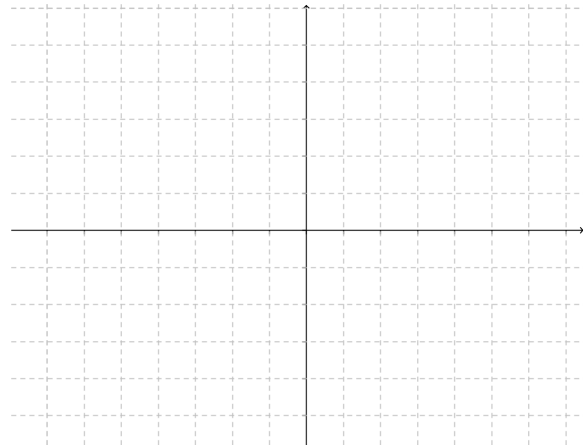
b. Write the system of inequalities that describe the region.

c. Name a vertical line that passes through the region.



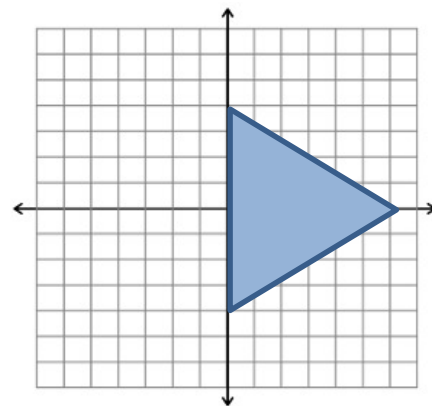
Example 2

Draw the triangular region in the plane given by the triangle with vertices $(0, 0)$, $(1, 3)$, and $(2, 1)$. Can we write a set of inequalities that describes this region?

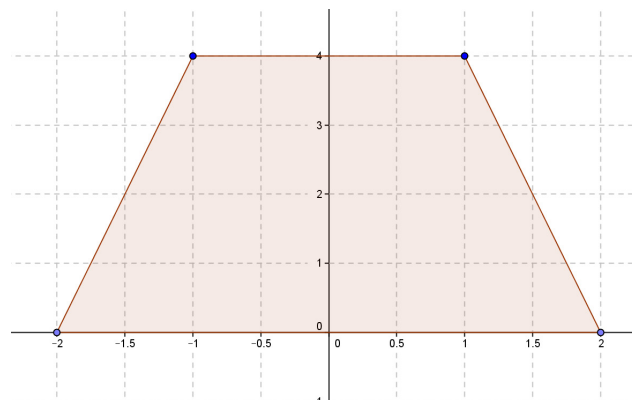


Exercises 4–5

4. Given the triangular region shown, describe this region with a system of inequalities.



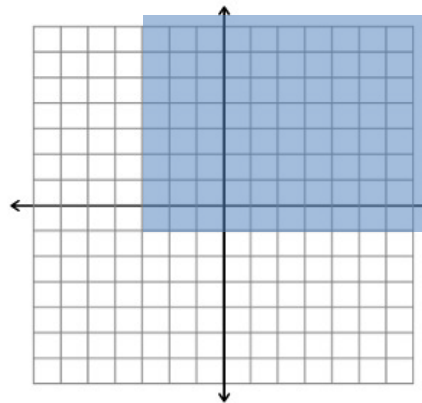
5. Given the trapezoid with vertices $(-2, 0)$, $(-1, 4)$, $(1, 4)$, and $(2, 0)$, describe this region with a system of inequalities.



Problem Set

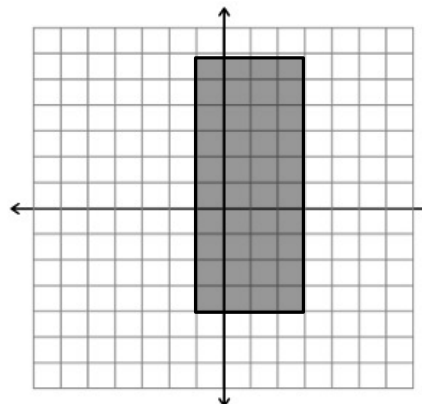
1. Given the region shown:

- How many half-planes intersect to form this region?
- Name three points on the boundary of the region.
- Describe the region in words.



2. Region T is shown to the right.

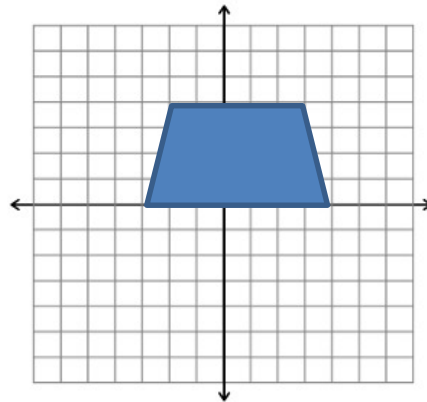
- Write the coordinates of the vertices.
- Write an inequality that describes the region.
- What is the length of the diagonals?
- Give the coordinates of a point that is both in the region and on one of the diagonals.



3. Jack wants to plant a garden in his back yard. His yard is 120 feet wide and 80 feet deep. He wants to plant a garden that is 20 feet by 30 feet.

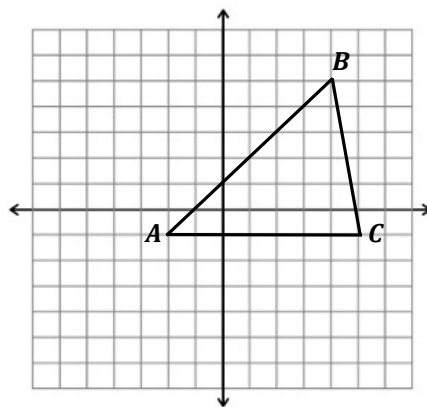
- Set up a grid for the backyard and place the garden on the grid. Explain why you placed your garden in its place on the grid.
- Write a system of inequalities to describe the garden.
- Write the equation of three lines that would go through the region that he could plant on, and explain your choices.

4. Given the trapezoidal region shown to the right:
- Write the system of inequalities describing the region.
 - Translate the region to the right 3 units and down 2 units. Write the system of inequalities describing the translated region.

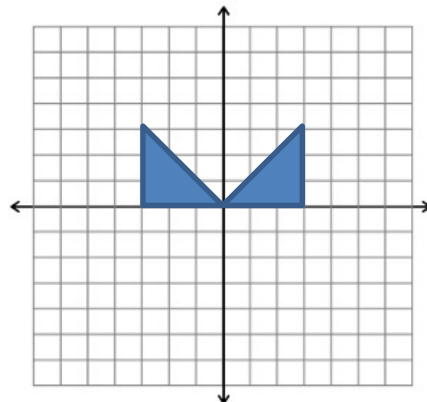


Challenge Problems:

5. Given the triangular region shown with vertices $A(-2, -1)$, $B(4, 5)$, and $C(5, -1)$:
- Describe the systems of inequalities that describe the region enclosed by the triangle.
 - Rotate the region 90° counterclockwise about Point A . How will this change the coordinates of the vertices?
 - Write the system of inequalities that describe the region enclosed in the rotated triangle.



6. Write a system of inequalities for the region shown.

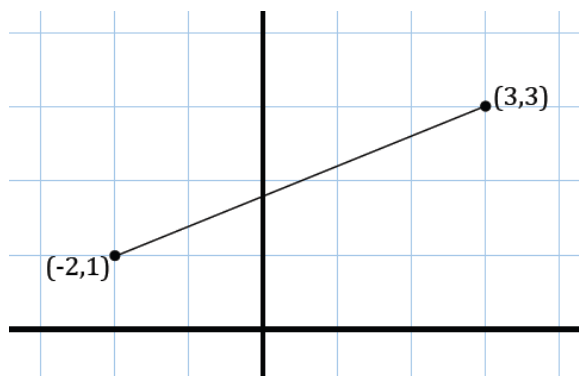


Lesson 3: Lines That Pass Through Regions

Classwork

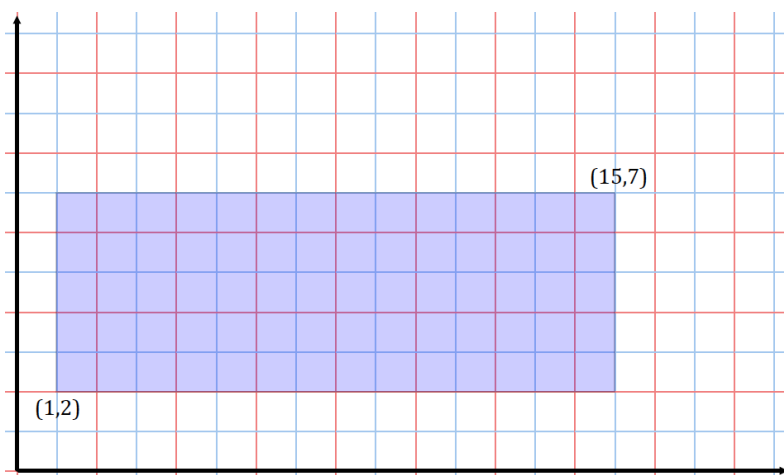
Opening Exercise

How can we use the Pythagorean theorem to find the length of \overline{AB} , or in other words, the distance between $A(-2,1)$ and $B(3,3)$? Find the distance between A and B .



Example 1

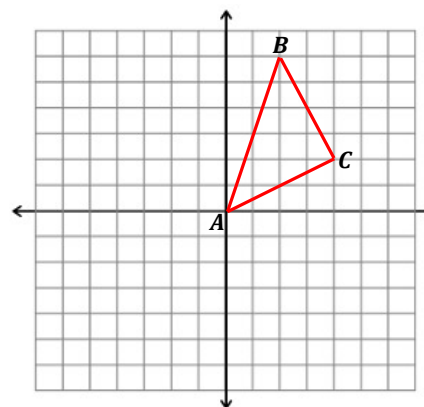
Consider the rectangular region:



- a. Does a line of slope 2 passing through the origin intersect this rectangular region? If so, which boundary points of the rectangle does it intersect? Explain how you know.
- b. Does a line of slope $\frac{1}{2}$ passing through the origin intersect this rectangular region? If so, which boundary points of the rectangle does it intersect?
- c. Does a line of slope $\frac{1}{3}$ passing through the origin intersect this rectangular region? If so, which boundary points of the rectangle does it intersect?
- d. A line passes through the lower right vertex of the rectangle? Does the line pass through the interior of the rectangular region or the boundary of the rectangular region? Does the line pass through both?
- e. For which values of m would a line of slope m through the origin intersect this region?
- f. For which values of m would a line of slope m through the point $(0, 1)$ intersect this region?

Example 2

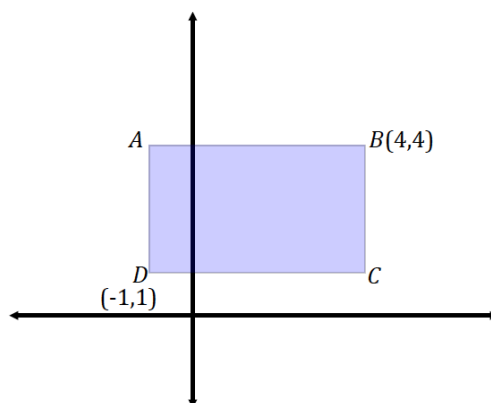
Consider the triangular region in the plane given by the triangle with vertices $A(0, 0)$, $B(2, 6)$, and $C(4, 2)$.



- The horizontal line $y = 2$ intersects this region. What are the coordinates of the two boundary points it intersects? What is the length of the horizontal segment within the region along this line?
- Graph the line $3x - 2y = 5$. Find the points of intersection with the triangular region and label them as X and Y .
- What is the length of the segment \overline{XY} ?
- A robot starts at position $(1, 3)$ and moves vertically downward towards the x -axis at a constant speed of 0.2 units per second. When will it hit the lower boundary of the triangular region that falls in its vertical path?

Exercise

Consider the given rectangular region:



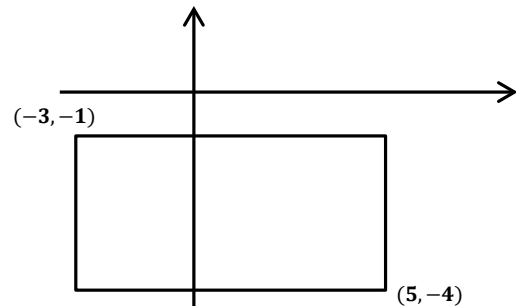
- Draw lines that pass through the origin and through each of the vertices of the rectangular region. Do each of the four lines cross multiple points in the region? Explain.
- Write the equation of a line that does not intersect the rectangular region at all.
- A robot is positioned at D and begins to move in a straight line with slope $m = 1$. When it intersects with a boundary, it then reorients itself and begins to move in a straight line with a slope of $m = -\frac{1}{2}$. What is the location of the next intersection the robot makes with the boundary of the rectangular region?
- What is the approximate distance of the robot's path in part (c)?

Problem Set

1. A line intersects a triangle at least once, but not at any of its vertices. What is the maximum number of sides that a line can intersect a triangle? Similarly, a square? A convex quadrilateral? A quadrilateral, in general?

2. Consider the rectangular region:

- What boundary points does a line through the origin with a slope of -2 intersect? What is the length of the segment within this region along this line?
- What boundary points does a line through the origin with a slope of 3 intersect? What is the length of the segment within this region along this line?
- What boundary points does a line through the origin with a slope of $-\frac{1}{5}$ intersect?
- What boundary points does a line through the origin with a slope of $\frac{1}{4}$ intersect?



3. Consider the triangular region in the plane given by the triangle $(-1, 3)$, $(1, -2)$, and $(-3, -3)$.
- The horizontal line $y = 1$ intersects this region. What are the coordinates of the two boundary points it intersects? What is the length of the horizontal segment within the region along this line?
 - What is the length of the section of the line $2x + 3y = -4$ that lies within this region?
 - If a robot starts at $(1, -2)$ and moves vertically downward at a constant speed of 0.75 units per second, when will it hit the lower boundary of the triangular region?
 - If the robot starts at $(1, -2)$ and moves horizontally left at a constant speed of 0.6 units per second, when will it hit the left boundary of the triangular region?
4. A computer software exists so that the cursor of the program begins and ends at the origin of the plane. A program is written to draw a triangle with vertices $A(1, 4)$, $B(6, 2)$, and $C(3, 1)$ so that the cursor only moves in straight lines and travels from the origin to A , then to B , then to C , then to A , and then back “home” to the origin.
- Sketch the cursor’s path, i.e., sketch the entire path from when it begins until when it returns “home.”
 - What is the approximate total distance traveled by the cursor?
 - Assume the cursor is positioned at B and is moving horizontally towards the y -axis at $\frac{2}{3}$ units per second. How long will it take to reach the boundary of the triangle?
5. An equilateral triangle with side length 1 is placed in the first quadrant so that one of its vertices is at the origin and another vertex is on the x -axis. A line passes through the point half the distance between the endpoints on one side and half the distance between the endpoints on the other side.
- Draw a picture that satisfies these conditions.
 - Find the equation of the line that you drew.

Lesson 4: Designing a Search Robot to Find a Beacon

Classwork

Opening Exercise

Write the equation of the line that satisfies the following conditions:

a. Has a slope of $m = -\frac{1}{4}$ and passes through the point $(0, -5)$.

b. Passes through the points $(1, 3)$ and $(-2, -1)$.

Exploratory Challenge

A search robot is sweeping through a flat plane in search of the homing beacon that is admitting a signal. (A homing beacon is a tracking device that sends out signals to identify the location). Programmers have set up a coordinate system so that their location is the origin, the positive x -axis is in the direction of east, and the positive y -axis is in the direction of north. The robot is currently 600 units south of the programmers' location and is moving in an approximate northeast direction along the line $y = 3x - 600$.

Along this line, the robot hears the loudest "ping" at the point $(400, 600)$. It detects this ping coming from approximately a southeast direction. The programmers have the robot return to the point $(400, 600)$. What is the equation of the path the robot should take from here to reach the beacon?

Begin by sketching the location of the programmers and the path traveled by the robot on graph paper; then, shade the general direction the ping is coming from.

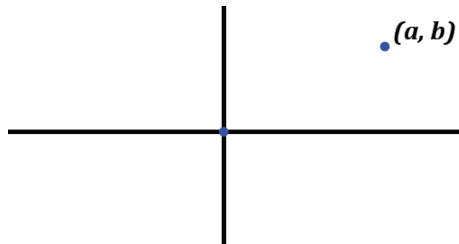
Notes:**Example 1**

The line segment connecting $(3, 7)$ to $(10, 1)$ is rotated counterclockwise 90° about the point $(3, 7)$.

- a. Plot the points.
- b. Where will the rotated endpoint land?
- c. Now rotate the original segment 90° clockwise. Before using a sketch, predict the coordinates of the rotated endpoint using what you know about the perpendicular slope of the rotated segment.

Exercise 1

The point (a, b) is labeled below:



- Using a and b , describe the location of (a, b) after a 90° counterclockwise about the origin. Draw a rough sketch to justify your answer.
- If the rotation was clockwise about the origin, what is the rotated location of (a, b) in terms of a and b ? Draw a rough sketch to justify your answer.
- What is the slope of the line through the origin and (a, b) ? What is the slope of the perpendicular line through the origin?
- What do you notice about the relationship between the slope of the line through the origin and (a, b) and the slope of the perpendicular line?

Problem Set

- Find the new coordinates of point $(0, 4)$ if it rotates:
 - 90° counterclockwise.
 - 90° clockwise.
 - 180° counterclockwise.
 - 270° clockwise.
- What are the new coordinates of the point $(-3, -4)$ if it is rotated about the origin:
 - Counterclockwise 90° ?
 - Clockwise 90° ?
- Line segment \overline{ST} connects points $S(7,1)$ and $T(2,4)$.
 - Where does point T land if the segment is rotated 90° counterclockwise about S ?
 - Where does point T land if the segment is rotated 90° clockwise about S ?
 - What is the slope of the original segment?
 - What is the slope of the rotated segments?
- Line segment \overline{VW} connects points $V(1,0)$ and $W(5,-3)$.
 - Where does point W land if the segment is rotated 90° counterclockwise about V ?
 - Where does point W land if the segment is rotated 90° clockwise about V ?
 - Where does point V land if the segment is rotated 90° counterclockwise about W ?
 - Where does point V land if the segment is rotated 90° clockwise about W ?
- If the slope of a line is 0, what is the slope of a line perpendicular to it? If the line has slope 1, what is the slope of a line perpendicular to it?
- If a line through the origin has a slope of 2, what is the slope of the line through the origin that is perpendicular to it?
- A line through the origin has a slope of $\frac{1}{3}$. Carlos thinks the slope of a perpendicular line at the origin will be 3. Do you agree? Explain why or why not.
- Could a line through the origin perpendicular to a line through the origin with slope $\frac{1}{2}$ pass through the point $(-1, 4)$? Explain how you know.

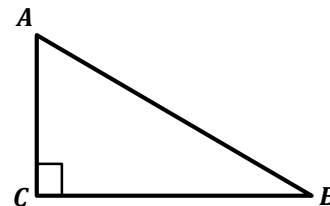
Lesson 5: Criterion for Perpendicularity

Classwork

Opening Exercise

In right triangle ABC , find the missing side.

- a. If $AC = 9$ and $CB = 12$, what is AB ? Explain how you know.



- b. If $AC = 5$ and $AB = 13$, what is CB ?

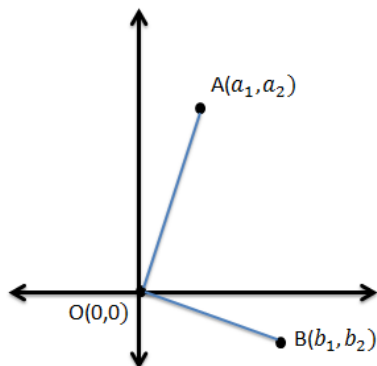
- c. If $AC = CB$ and $AB = 2$, what is AC (and CB)?

Exercise 1

1. Use the grid at the right.
- a. Plot points $O(0,0)$, $P(3,-1)$, and $Q(2,3)$ on the coordinate plane.
- b. Determine whether \overline{OP} and \overline{OQ} are perpendicular. Support your findings.



Example 2



Exercises 2–4

2. Given points $A(6,4)$, $B(24,-6)$, $C(1,4)$, $P(2,-3)$, $S(-18,-12)$, $T(-3,-12)$, $U(-8,2)$, and $W(-6,9)$, find all pairs of segments from the list below that are perpendicular. Support your answer.

\overline{OA} , \overline{OB} , \overline{OC} , \overline{OP} , \overline{OS} , \overline{OT} , \overline{OU} , and \overline{OW}

3. The points $O(0,0)$, $A(-4,1)$, $B(-3,5)$, and $C(1,4)$ are the vertices of parallelogram $OABC$. Is this parallelogram a rectangle? Support your answer.



Problem Set

1. Prove using the Pythagorean theorem that \overline{AC} is perpendicular to \overline{AB} given $A(-2, -2)$, $B(5, -2)$, and $C(-2, 22)$.
2. Using the general formula for perpendicularity of segments through the origin and $(90, 0)$, determine if segments \overline{OA} and \overline{OB} are perpendicular.
 - a. $A(-3, -4)$, $B(4, 3)$
 - b. $A(8, 9)$, $B(18, -16)$
3. Given points $O(0, 0)$, $S(2, 7)$, and $T(7, -2)$, where \overline{OS} is perpendicular to \overline{OT} , will the images of the segments be perpendicular if the three points O , S , and T are translated four units to the right and eight units up? Explain your answer.
4. In Example 1, we saw that \overline{OA} was perpendicular to \overline{OB} for $O(0, 0)$, $A(6, 4)$, and $B(-2, 3)$. Suppose $P(5, 5)$, $Q(11, 9)$, and $R(3, 8)$. Are segments \overline{PQ} and \overline{PR} perpendicular? Explain without using triangles or the Pythagorean theorem.
5. Challenge: Using what we learned in Exercise 2, if $C(c_1, c_2)$, $A(a_1, a_2)$, and $B(b_1, b_2)$, what is the general condition of a_1 , a_2 , b_1 , b_2 , c_1 , and c_2 that ensures segments \overline{CA} and \overline{CB} are perpendicular?
6. A robot that picks up tennis balls is on a straight path from $(8, 6)$ towards a ball at $(-10, -5)$. The robot picks up a ball at $(-10, -5)$, then turns 90° right. What are the coordinates of a point that the robot can move towards to pick up the last ball?
7. Gerry thinks that the points $(4, 2)$ and $(-1, 4)$ form a line perpendicular to a line with slope 4. Do you agree? Why or why not?

Lesson 6: Segments That Meet at Right Angles

Classwork

Opening Exercise

Carlos thinks that the segment having endpoints $A(0,0)$ and $B(6,0)$ is perpendicular to the segment with endpoints $A(0,0)$ and $C(-2,0)$. Do you agree? Why or why not?

Working with a partner, given $A(0,0)$ and $B(3,-2)$, find the coordinates of a point C so that $\overline{AC} \perp \overline{AB}$.

Example 1

Given points $A(2,2)$, $B(10,16)$, $C(-3,1)$, and $D(4,-3)$, are segments \overline{AB} and \overline{CD} perpendicular? Are the lines containing the segments perpendicular? Explain.

Exercises 1–4

1. Given $A(a_1, a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$, and $D(d_1, d_2)$, find a general formula in terms of $a_1, a_2, b_1, b_2, c_1, c_2, d_1$, and d_2 that will let us determine whether segments \overline{AB} and \overline{CD} are perpendicular.

2. Recall the Opening Exercise of Lesson 4 in which a robot is traveling along a linear path given by the equation $y = 3x - 600$. The robot hears a ping from a homing beacon when it reaches the point $B(400, 600)$ and turns to travel along a linear path given by the equation $y - 600 = -\frac{1}{3}(x - 400)$. If the homing beacon lies on the x -axis, what is its exact location? (Use your own graph paper to visualize the scenario.)
 - a. If point A is the y -intercept of the original equation, what are the coordinates of point A ?
 - b. What are the endpoints of the original segment of motion?
 - c. If the beacon lies on the x -axis, what is the y -value of this point, C ?
 - d. Translate point B to the origin. What are the coordinates of A' , B' , and C' ?
 - e. Use the formula derived in this lesson to determine the coordinates of point C .

3. A triangle in the coordinate plane has vertices $A(0, 10)$, $B(-8, 8)$, and $C(-3, 5)$. Is it a right triangle? If so, at which vertex is the right angle? (Hint: Plot the points and draw the triangle on a coordinate plane to help you determine which vertex is the best candidate for the right angle.)
4. $A(-7, 1)$, $B(-1, 3)$, $C(5, -5)$, and $D(-5, -5)$ are vertices of a quadrilateral. If \overline{AC} bisects \overline{BD} , but \overline{BD} does not bisect \overline{AC} , determine whether $ABCD$ is a kite.

Problem Set

- Are the segments through the origin and the points listed perpendicular? Explain.
 - $A(9, 10)$, $B(10, 9)$
 - $C(9, 6)$, $D(4, -6)$
- Given $M(5, 2)$, $N(1, -4)$, and L listed below, are segments \overline{LM} and \overline{MN} perpendicular? Translate M to the origin, write the coordinates of the images of the points, then explain without using slope.
 - $L(-1, 6)$
 - $L(11, -2)$
 - $L(9, 8)$
- Is triangle PQR , where $P(-7, 3)$, $Q(-4, 7)$, and $R(1, -3)$, a right triangle? If so, which angle is the right angle? Justify your answer.
- A quadrilateral has vertices $(2 + \sqrt{2}, -1)$, $(8 + \sqrt{2}, 3)$, $(6 + \sqrt{2}, 6)$, and $(\sqrt{2}, 2)$. Prove that the quadrilateral is a rectangle.
- Given points $G(-4, 1)$, $H(3, 2)$, and $I(-2, -3)$, find the x -coordinate of point J with y -coordinate 4 so that the lines \overleftrightarrow{GH} and \overleftrightarrow{IJ} are perpendicular.
- A robot begins at position $(-80, 45)$ and moves on a path to $(100, -60)$. It turns 90° counterclockwise.
 - What point with y -coordinate 120 is on this path?
 - Write an equation of the line after the turn.
 - If it stops to charge on the x -axis, what is the location of the charger?
- Determine the missing vertex of a right triangle with vertices $(6, 2)$ and $(5, 5)$ if the third vertex is on the y -axis. Verify your answer by graphing.
- Determine the missing vertex for a rectangle with vertices $(3, -2)$, $(5, 2)$, and $(-1, 5)$, and verify by graphing. Then, answer the questions that follow.
 - What is the length of the diagonal?
 - What is a point on both diagonals in the interior of the figure?
- A right triangle has vertices $(1, 3)$ and $(6, -1)$ and a third vertex located in Quadrant IV.
 - Determine the coordinates of the missing vertex.
 - Reflect the triangle across the y -axis. What are the new vertices?
 - If the original triangle is rotated 90° counterclockwise about the vertex $(6, -1)$, what are the coordinates of the other vertices?
 - Now rotate the original triangle 90° clockwise about $(6, -1)$. What are the coordinates of the other vertices?
 - What do you notice about both sets of vertices? Explain what you observe.

Lesson 7: Equations for Lines Using Normal Segments

Classwork

Opening Exercise

The equations given are in standard form. Put each equation in slope-intercept form. State the slope and the y -intercept.

1. $6x + 3y = 12$

2. $5x + 7y = 14$

3. $2x - 5y = -7$

Example 1

Given $A(5, -7)$ and $B(8, 2)$:

a. Find an equation for the line through A and perpendicular to \overline{AB} .

b. Find an equation for the line through B and perpendicular to \overline{AB} .

Exercises 1–2

1. Given $U(-4, -1)$ and $V(7, 1)$:
 - a. Write an equation for the line through U and perpendicular to \overline{UV} .
 - b. Write an equation for the line through V and perpendicular to \overline{UV} .

2. Given $S(5, -4)$ and $T(8, 12)$:
 - a. Write an equation for the line through S and perpendicular to \overline{ST} .
 - b. Write an equation for the line through T and perpendicular to \overline{ST} .

Closing

Describe the characteristics of a normal segment.

Every equation of a line through a given point (a, b) has the form $A(x - a) + B(y - b) = 0$. Explain how the values of A and B are obtained.

Problem Set

1. Given points $C(-4,3)$ and $D(3,3)$:
 - a. Write the equation of the line through C and perpendicular to \overline{CD} .
 - b. Write the equation of the line through D and perpendicular to \overline{CD} .
2. Given points $N(7,6)$ and $M(7,-2)$:
 - a. Write the equation of the line through M and perpendicular to \overline{MN} .
 - b. Write the equation of the line through N and perpendicular to \overline{MN} .
3. The equation of a line is given by the equation $8(x - 4) + 3(y + 2) = 0$.
 - a. What are the coordinates of the image of the endpoint of the normal segment that does not lie on the line? Explain your answer.
 - b. What translation occurred to move the point of perpendicularity to the origin?
 - c. What were the coordinates of the original point of perpendicularity? Explain your answer.
 - d. What were the endpoints of the original normal segment?
4. A coach is laying out lanes for a race. The lanes are perpendicular to a segment of the track such that one endpoint of the segment is $(2, 50)$ and the other is $(20, 65)$. What are the equations of the lines through the endpoints?

Lesson 8: Parallel and Perpendicular Lines

Classwork

Exercise 1

1.
 - a. Write an equation of the line that passes through the origin that intersects the line $2x + 5y = 7$ to form a right angle.
 - b. Determine whether the lines given by the equations $2x + 3y = 6$ and $y = \frac{3}{2}x + 4$ are perpendicular. Support your answer.
 - c. Two lines having the same y -intercept are perpendicular. If the equation of one of these lines is $y = -\frac{4}{5}x + 6$, what is the equation of the second line?

Example 2

- a. What is the relationship between two coplanar lines that are perpendicular to the same line?

- b. Given two lines, l_1 and l_2 , with equal slopes and a line k that is perpendicular to one of these two parallel lines, l_1 :
- What is the relationship between line k and the other line, l_2 ?
 - What is the relationship between l_1 and l_2 ?

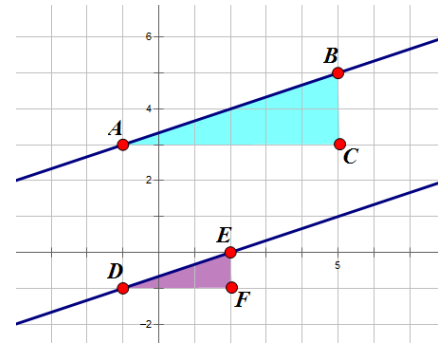
Exercises 2–7

2. Given a point $(-3, 6)$ and a line $y = 2x - 8$:
- What is the slope of the line?
 - What is the slope of any line parallel to the given line?
 - Write an equation of a line through the point and parallel to the line.
 - What is the slope of any line perpendicular to the given line? Explain.

3. Find an equation of a line through $(0, -7)$ and parallel to the line $y = \frac{1}{2}x + 5$.
- What is the slope of any line parallel to the given line? Explain your answer.
 - Write an equation of a line through the point and parallel to the line.
 - If a line is perpendicular to $y = \frac{1}{2}x + 5$, will it be perpendicular to $x - 2y = 14$? Explain.
4. Find an equation of a line through $(\sqrt{3}, \frac{1}{2})$ parallel to the line:
- $x = -9$
 - $y = -\sqrt{7}$
 - What can you conclude about your answer in parts (a) and (b)?
5. Find an equation of a line through $(-\sqrt{2}, \pi)$ parallel to the line $x - 7y = \sqrt{5}$.

6. Recall that our search robot is moving along the line $y = 3x - 600$ and wishes to make a right turn at the point $(400, 600)$. Find an equation for the perpendicular line on which the robot is to move. Verify that your line intersects the x -axis at $(2200, 0)$.
7. A robot, always moving at a constant speed of 2 feet per second, starts at position $(20, 50)$ on the coordinate plane and heads in a south-east direction along the line $3x + 4y = 260$. After 15 seconds, it turns left 90° and travels in a straight line in this new direction.
- What are the coordinates of the point at which the robot made the turn? What might be a relatively straightforward way of determining this point?
 - Find an equation for the second line on which the robot traveled.
 - If, after turning, the robot travels for 20 seconds along this line and then stops, how far will it be from its starting position?
 - What is the equation of the line the robot needs to travel along in order to now return to its starting position? How long will it take for the robot to get there?

Problem Set

- Write the equation of the line through $(-5, 3)$ and:
 - Parallel to $x = -1$.
 - Perpendicular to $x = -1$.
 - Parallel to $y = \frac{3}{5}x + 2$.
 - Perpendicular to $y = \frac{3}{5}x + 2$.
- Write the equation of the line through $(\sqrt{3}, \frac{5}{4})$ and:
 - Parallel to $y = 7$.
 - Perpendicular to $y = 7$.
 - Parallel to $\frac{1}{2}x - \frac{3}{4}y = 10$.
 - Perpendicular to $\frac{1}{2}x - \frac{3}{4}y = 10$.
- A vacuum robot is in a room and charging at position $(0, 5)$. Once charged, it begins moving on a northeast path at a constant speed of $\frac{1}{2}$ foot per second along the line $4x - 3y = -15$. After 60 seconds, it turns right 90° and travels in the new direction.
 - What are the coordinates of the point at which the robot made the turn?
 - Find an equation for the second line on which the robot traveled.
 - If after turning, the robot travels 80 seconds along this line, how far has it traveled from its starting position?
 - What is the equation of the line the robot needs to travel along in order to return and recharge? How long will it take the robot to get there?
- Given the statement \overleftrightarrow{AB} is parallel to \overleftrightarrow{DE} , construct an argument for or against this statement using the two triangles shown.
 
- Recall the proof we did in Example 1: Let l_1 and l_2 be two non-vertical lines in the Cartesian plane. l_1 and l_2 are perpendicular if and only if their slopes are negative reciprocals of each other. In class, we looked at the case where both y-intercepts were not zero. In Lesson 5, we looked at the case where both y-intercepts were equal to zero, when the vertex of the right angle was at the origin. Reconstruct the proof for the case where one line has a y-intercept of zero, and the other line has a non-zero y-intercept.
- Challenge: Reconstruct the proof we did in Example 1 if one line has a slope of zero.

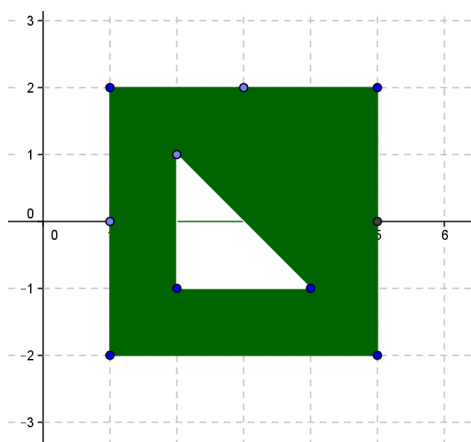
Lesson 9: Perimeter and Area of Triangles in the Cartesian Plane

Classwork

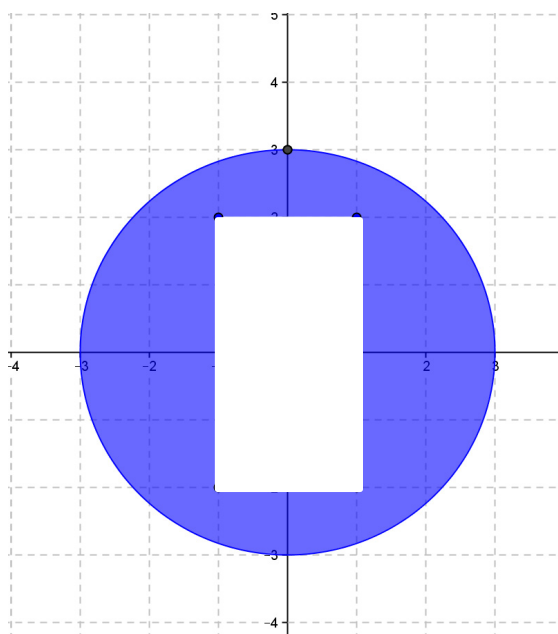
Opening Exercises

Find the area of the shaded region.

a.



b.

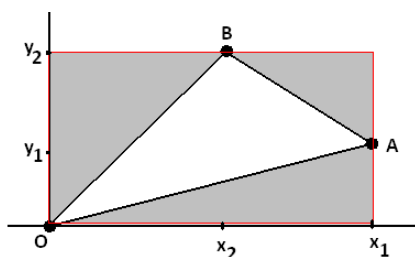


Example 1

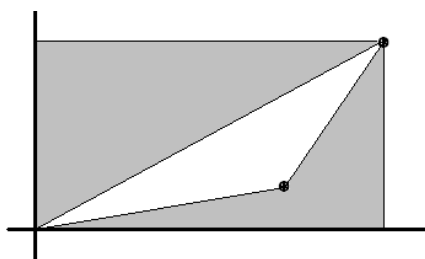
Consider a triangular region in the plane with vertices $O(0, 0)$, $A(5, 2)$, and $B(3, 4)$. What is the perimeter of the triangular region?

What is the area of the triangular region?

Find the general formula for the area of the triangle with vertices $O(0, 0)$, $A(x_1, y_1)$, and $B(x_2, y_2)$ as shown.



Does the formula work for this triangle?



Exercise 1

Find the area of the triangles with vertices listed, first by finding the area of the rectangle enclosing the triangle and subtracting the area of the surrounding triangles, then by using the formula $\frac{1}{2}(x_1y_2 - x_2y_1)$.

a. $O(0, 0), A(5, 6), B(4, 1)$

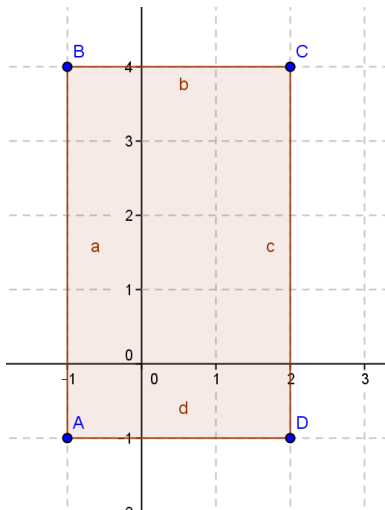
b. $O(0, 0), A(3, 2), B(-2, 6)$

c. $O(0, 0), A(5, -3), B(-2, 6)$

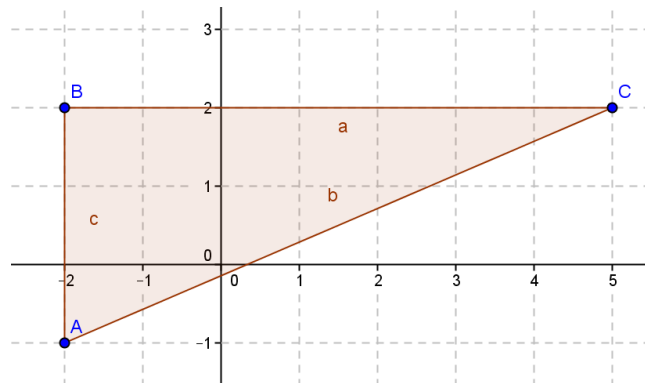
Problem Set

1. Use coordinates to compute the perimeter and area of each polygon.

a.

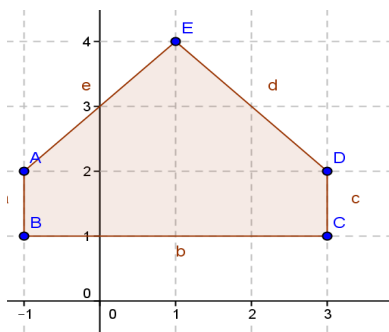


b.

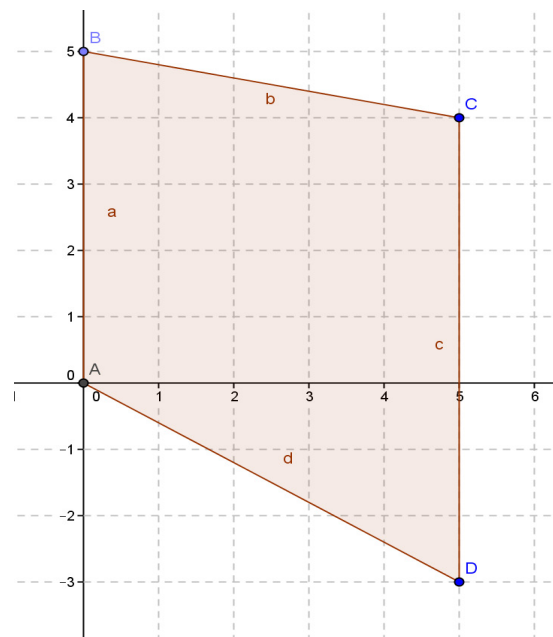


2. Given the figures below, find the area by decomposing into rectangles and triangles.

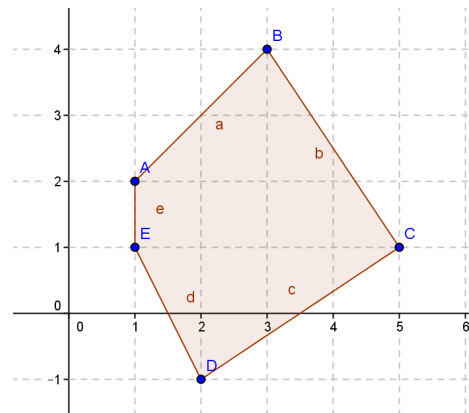
a.



b.

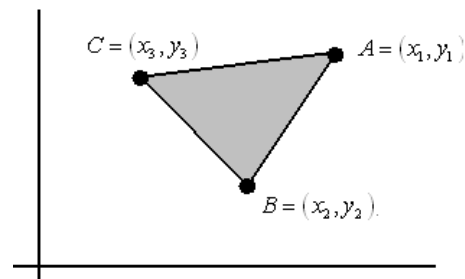


3. Challenge: Find the area by decomposing the given figure into triangles.

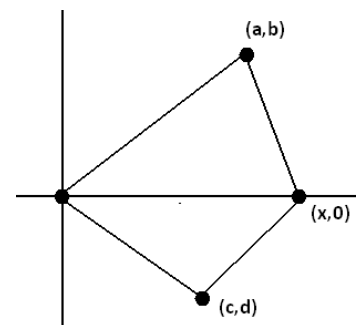


4. When using the shoelace formula to work out the area of $\triangle ABC$, we have some choices to make. For example, we can start at any one of the three vertices A , B , or C , and we can move either in a clockwise or counterclockwise direction. This gives six options for evaluating the formula.

Show that the shoelace formula obtained is identical for the three options that move in a clockwise direction (A to C to B or C to B to A or B to A to C) and identical for the three options in the reverse direction. Verify that the two distinct formulas obtained differ only by a minus sign.



5. Suppose two triangles share a common edge. By translating and rotating the triangles, we can assume that the common edge lies along the x -axis with one endpoint at the origin.
- Show that if we evaluate the shoelace formula for each triangle, both calculated in the same clockwise direction, then the answers are both negative.
 - Show that if we evaluate them both in a counter-clockwise direction, then both are positive.
 - Explain why evaluating one in one direction and the second in the opposite direction, the two values obtained are opposite in sign.



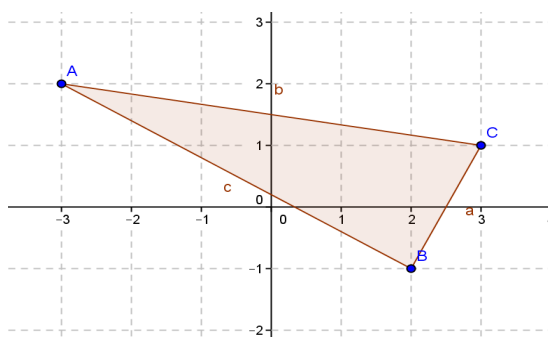
6. A textbook has a picture of a triangle with vertices $(3, 6)$ and $(5, 2)$. Something happened in printing the book and the coordinates of the third vertex are listed as $(-1, \blacksquare)$. The answers in the back of the book give the area of the triangle as 6 square units.
- What is the y -coordinate of the third vertex?
 - What if both coordinates were missing, but the area was known. Could you use algebra to find the third coordinate? Explain.

Lesson 10: Perimeter and Area of Polygonal Regions in the Cartesian Plane

Classwork

Opening Exercise

Find the area of the triangle given. Compare your answer and method to your neighbor's and discuss differences.



Exercises 1–6

1. Given rectangle $ABCD$:

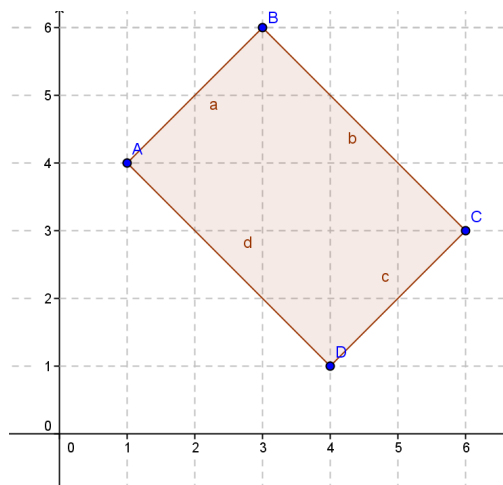
a. Identify the vertices.

b. Find the perimeter using the distance formula.

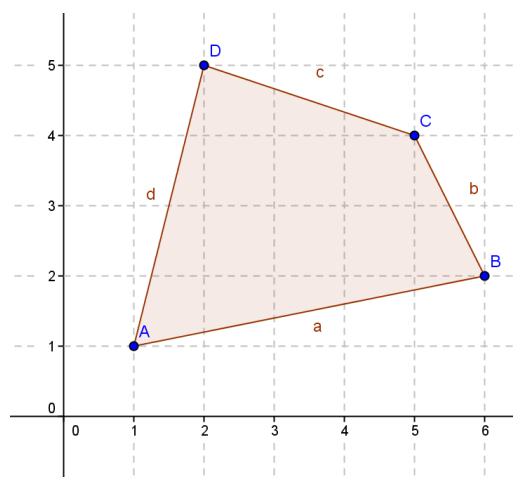
c. Find the area using the area formula.

d. List the vertices starting with A moving counterclockwise.

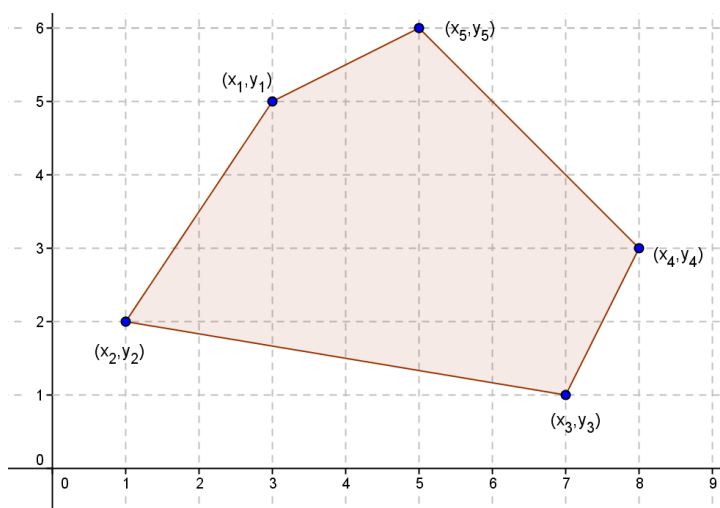
e. Verify the area using the shoelace formula.



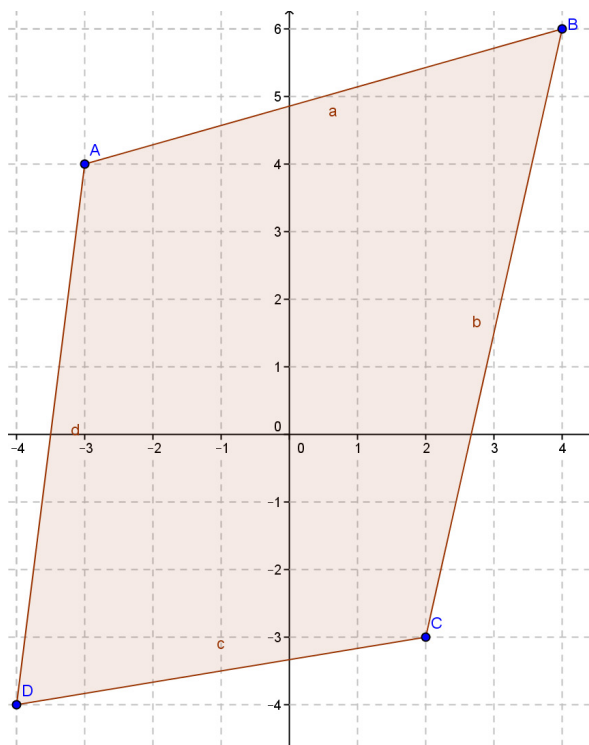
2. Calculate the area and perimeter of the given quadrilateral using the shoelace formula.



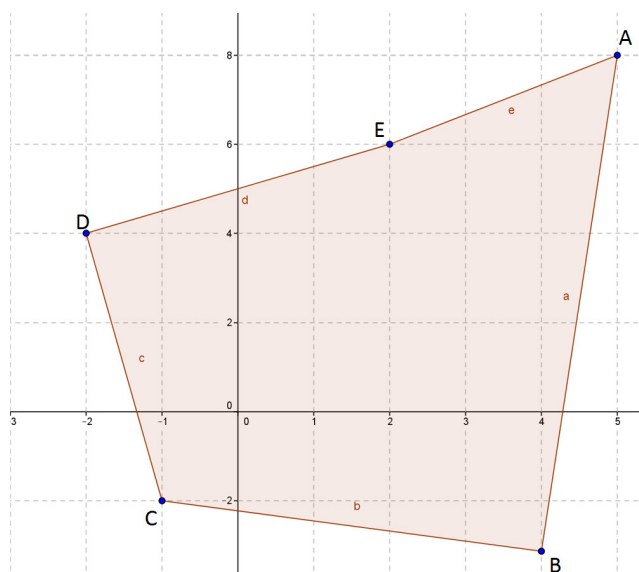
3. Break up the pentagon to find the area using Green's theorem. Compare your method with a partner.



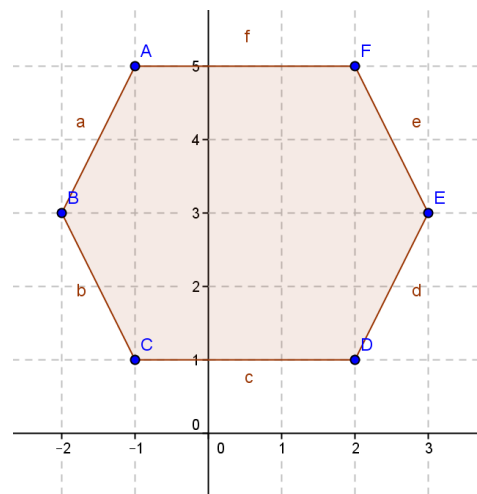
4. Find the perimeter and the area of the quadrilateral with vertices $A(-3, 4)$, $B(4, 6)$, $C(2, -3)$, and $D(-4, -4)$.



5. Find the area of the pentagon with vertices $A(5, 8)$, $B(4, -3)$, $C(-1, -2)$, $D(-2, 4)$, and $E(2, 6)$.

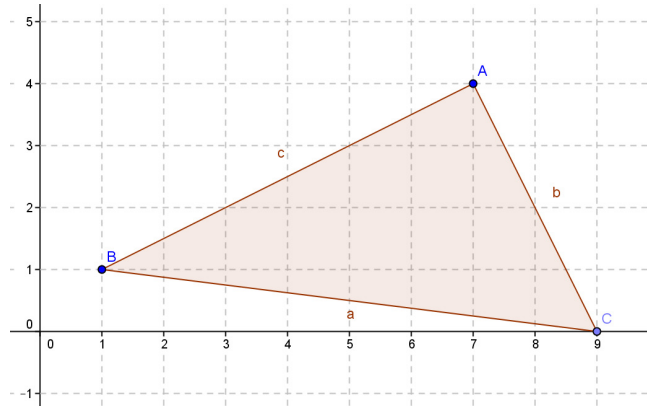


6. Find the area and perimeter of the hexagon shown.

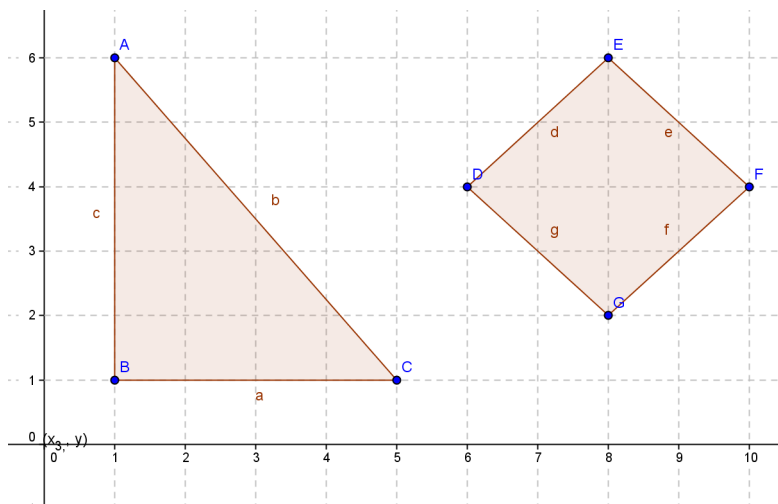


Problem Set

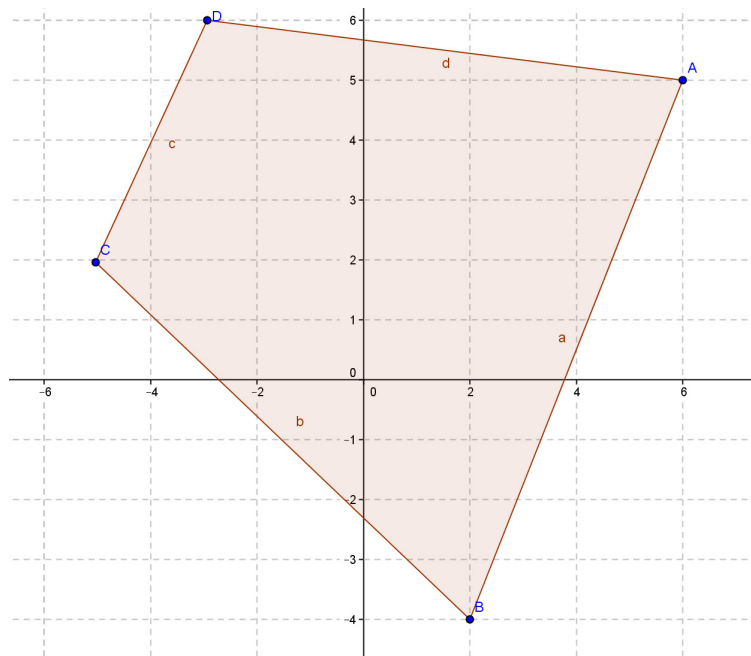
- Given triangle ABC with vertices $(7, 4)$, $(1, 1)$, and $(9, 0)$:



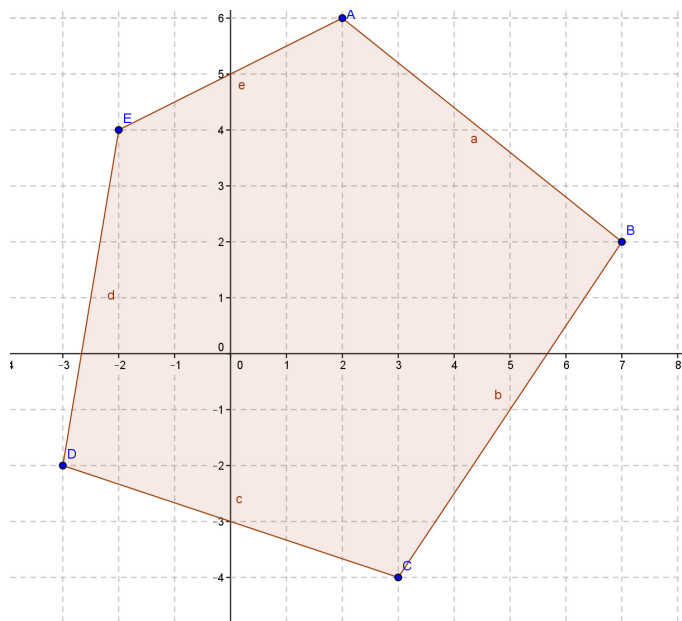
- Calculate the perimeter using the distance formula.
 - Calculate the area using the traditional area formula.
 - Calculate the area using the shoelace formula.
 - Explain why the shoelace formula might be more useful and efficient if you were just asked to find the area.
-
- Given triangle ABC and quadrilateral $DEFG$, describe how you would find the area of each and why you would choose that method, and then find the areas.



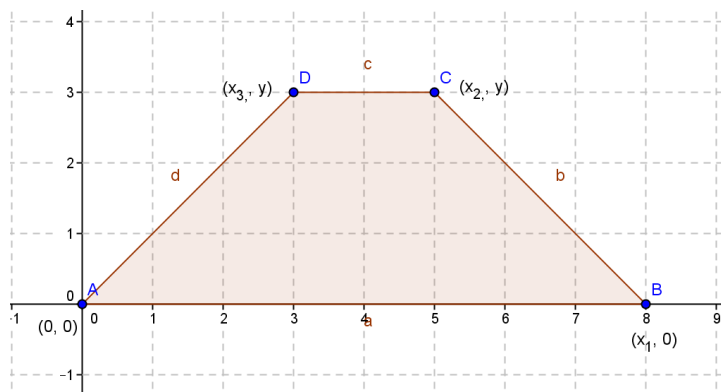
3. Find the area and perimeter of quadrilateral $ABCD$ with vertices $A(6, 5)$, $B(2, -4)$, $C(-5, 2)$, and $D(-3, 6)$.



4. Find the area and perimeter of pentagon $ABCDE$ with vertices $A(2, 6)$, $B(7, 2)$, $C(3, -4)$, $D(-3, -2)$, and $E(-2, 4)$.



5. Show that the shoelace formula (Green's theorem) used on the trapezoid shown confirms the traditional formula for the area of a trapezoid $\frac{1}{2}(b_1 + b_2) \cdot h$.



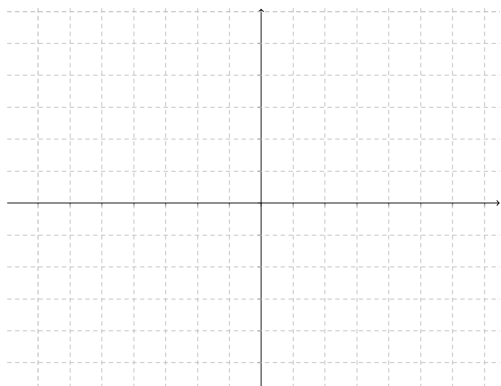
Lesson 11: Perimeters and Areas of Polygonal Regions Defined by Systems of Inequalities

Classwork

Opening Exercise

Graph the following:

a. $y \leq 7$



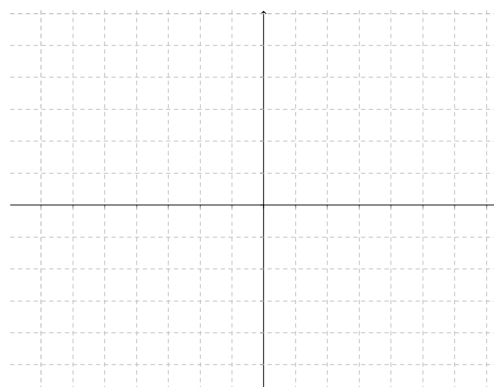
b. $x > -3$



c. $y < \frac{1}{2}x - 4$

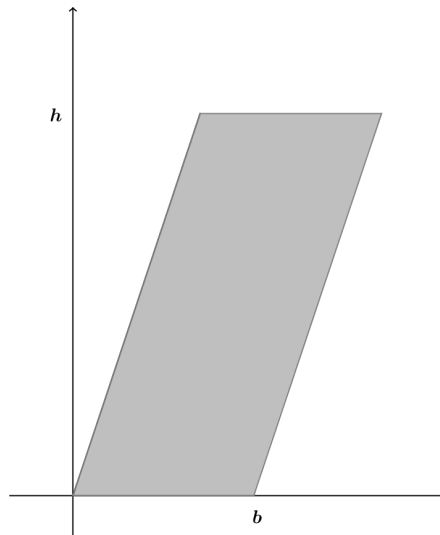


d. $y \geq -\frac{2}{3}x + 5$

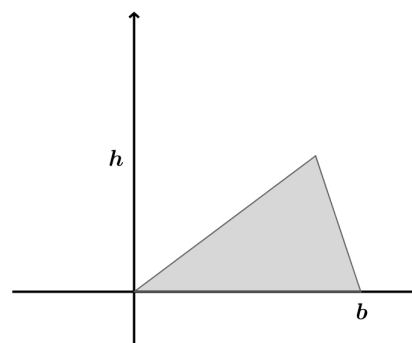


Example 1

A parallelogram with base of length b and height h can be situated in the coordinate plane as shown. Verify that the shoelace formula gives the area of the parallelogram as bh .

**Example 2**

A triangle with base b and height h can be situated in the coordinate plane as shown. According to Green's theorem, what is the area of the triangle?



Exercises 1–2

1. A quadrilateral region is defined by the system of inequalities below:

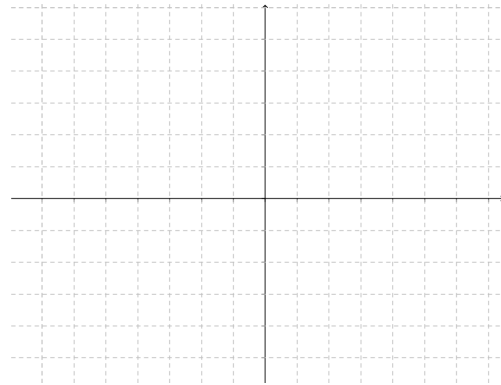
$$y \leq x + 6$$

$$y \leq -2x + 12$$

$$y \geq 2x - 4$$

$$y \geq -x + 2$$

- Sketch the region.
- Determine the vertices of the quadrilateral.
- Find the perimeter of the quadrilateral region.
- Find the area of the quadrilateral region.



2. A quadrilateral region is defined by the system of inequalities below:

$$y \leq x + 5$$

$$y \geq x - 4$$

$$y \leq 4$$

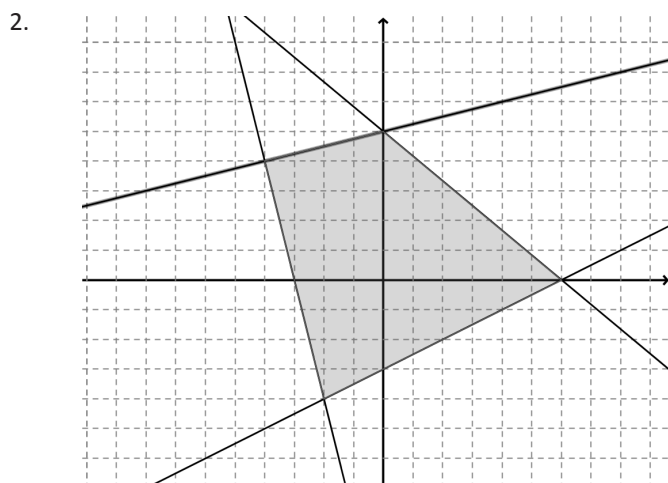
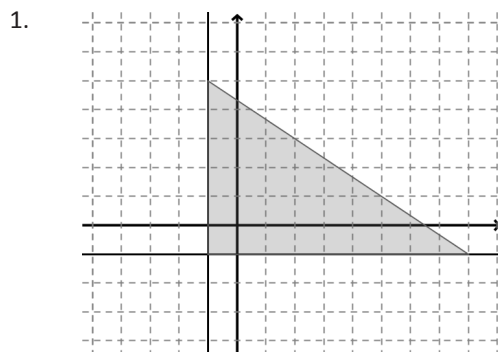
$$y \geq -\frac{5}{4}x - 4$$

- Sketch the region.
- Determine the vertices of the quadrilateral.
- Which quadrilateral is defined by these inequalities? How can you prove your conclusion?
- Find the perimeter of the quadrilateral region.
- Find the area of the quadrilateral region.



Problem Set

For Problems 1–2 below, identify the system of inequalities that defines the region shown.



For Problems 3–5 below, a triangular or quadrilateral region is defined by the system of inequalities listed.

- Sketch the region.
- Determine the coordinates of the vertices.
- Find the perimeter of the region rounded to the nearest hundredth if necessary.
- Find the area of the region rounded to the nearest tenth if necessary.

3. $8x - 9y \geq -22$ $x + y \leq 10$ $5x - 12y \leq -1$

4. $x + 3y \geq 0$ $4x - 3y \geq 0$ $2x + y \leq 10$

5. $2x - 5y \geq -14$ $3x + 2y \leq 17$ $2x - y \leq 9$ $x + y \geq 0$

Lesson 12: Dividing Segments Proportionately

Classwork

Exercises 1–4

1. Find the midpoint of \overline{ST} given $S(-2, 8)$ and $T(10, -4)$.

2. Find the point on the directed segment from $(-2, 0)$ to $(5, 8)$ that divides it in the ratio of 1:3.

3. Given \overline{PQ} and point R that lies on \overline{PQ} such that point R lies $\frac{7}{9}$ of the length of \overline{PQ} from point P along \overline{PQ} .
 - a. Sketch the situation described.

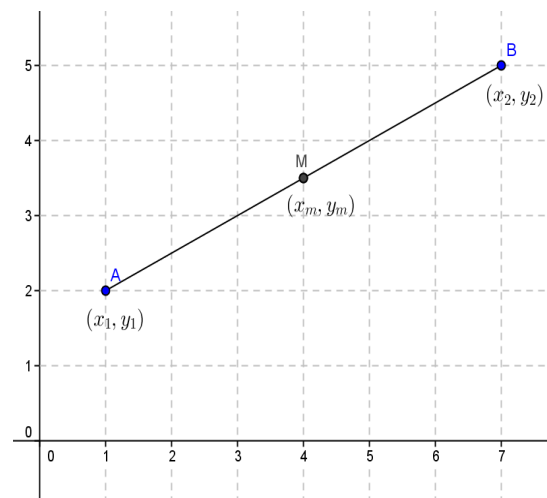
 - b. Is point R closer to P or closer to Q , and how do you know?

- c. Use the given information to determine the following ratios:
- i. $PR:PQ$
 - ii. $RQ:PQ$
 - iii. $PR:RQ$
 - iv. $RQ:PR$
- d. If the coordinates of point P are $(0,0)$ and the coordinates of point R are $(14,21)$, what are the coordinates of point Q ?
4. A robot is at position $A(40, 50)$ and is heading toward the point $B(2000, 2000)$ along a straight line at a constant speed. The robot will reach point B in 10 hours.
- a. What is the location of the robot at the end of the third hour?
 - b. What is the location of the robot five minutes before it reaches point B ?

- c. If the robot keeps moving along the straight path at the same constant speed as it passes through point B , what will be its location at the twelfth hour?
- d. Compare the value of the abscissa (x -coordinate) to the ordinate (y -coordinate) before, at, and after the robot passes point B ?
- e. Could you have predicted the relationship that you noticed in part (d) based on the coordinates of points A and B ?

Problem Set

- Given $F(0, 2)$ and $G(2, 6)$. If point S lies $\frac{5}{12}$ of the way along \overline{FG} , closer to F than to G , find the coordinates of S . Then verify that this point lies on \overline{FG} .
- Point C lies $\frac{5}{6}$ of the way along \overline{AB} , closer to B than to A . If the coordinates of point A are $(12, 5)$ and the coordinates of point C are $(9.5, -2.5)$, what are the coordinates of point B ?
- Find the point on the directed segment from $(-3, -2)$ to $(4, 8)$ that divides it into a ratio of 3: 2.
- A robot begins its journey at the origin, point O , and travels along a straight line path at a constant rate. Fifteen minutes into its journey the robot is at $A(35, 80)$.
 - If the robot does not change speed or direction, where will it be 3 hours into its journey (Call this point B)?
 - The robot continues past point B for a certain period of time until it has traveled an additional $\frac{3}{4}$ the distance it traveled in the first 3 hours and stops.
 - How long did the robot's entire journey take?
 - What is the robot's final location?
 - What was the distance the robot traveled in the last leg of its journey?
- Given \overline{LM} and point R that lies on \overline{LM} , identify the following ratios given that point R lies $\frac{a}{b}$ of the way along \overline{LM} , closer to L than to M .
 - $LR:LM$
 - $RM:LM$
 - $RL:RM$
- Given \overline{AB} with midpoint M as shown, prove that the point on the directed segment from A to B that divides \overline{AB} into a ratio of 1: 3 is the midpoint of \overline{AM} .



Lesson 13: Analytic Proofs of Theorems Previously Proved by Synthetic Means

Classwork

Opening Exercise

Let $A(30,40)$, $B(60,50)$, and $C(75, 120)$ be vertices of a triangle.

- Find the coordinates of the midpoint M of \overline{AB} and the point G_1 that is the point one-third of the way along \overline{MC} , closer to M than to C .
- Find the coordinates of the midpoint N of \overline{BC} and the point G_2 that is the point one-third of the way along \overline{NA} , closer to N than to A .
- Find the coordinates of the midpoint R of \overline{CA} and the point G_3 that is the point one-third of the way along \overline{RB} , closer to R than to B .

Exercise 1

- a. Given triangle ABC with vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$, find the coordinates of the point of concurrency.
- b. Let $A(-23, 12)$, $B(13, 36)$, and $C(23, -1)$ be vertices of a triangle. Where will the medians of this triangle intersect?

Exercise 2

Prove that the diagonal of a parallelogram bisect each other.

Problem Set

- Point M is the midpoint of segment \overline{AC} . Find the coordinates of M :
 - $A(2, 3), C(6, 10)$
 - $A(-7, 5), C(4, -9)$
- $M(-2, 10)$ is the midpoint of segment \overline{AB} . If A has coordinates $(4, -5)$, what are the coordinates of B ?
- Line A is the perpendicular bisector of segment \overline{BC} with $B(-2, -1)$ and $C(4, 1)$.
 - What is the midpoint of \overline{BC} ?
 - What is the slope of \overline{BC} ?
 - What is the slope of line A ? (Remember, it is perpendicular to \overline{BC} .)
 - Write the equation of line A , the perpendicular bisector of \overline{BC} .
- Find the coordinates of the intersection of the medians of $\triangle ABC$ given $A(-5, 3)$, $B(6, -4)$, and $C(10, 10)$.
- Use coordinates to prove that the diagonals of a parallelogram meet at the intersection of the segments that connect the midpoints of its opposite sides.
- Given a quadrilateral with vertices $E(0, 5)$, $F(6, 5)$, $G(4, 0)$, and $H(-2, 0)$:
 - Prove quadrilateral $EFGH$ is a parallelogram.
 - Prove $(2, 2.5)$ is a point on both diagonals of the quadrilateral.
- Prove quadrilateral $WXYZ$ with vertices $W(1, 3)$, $X(4, 8)$, $Y(10, 11)$, and $Z(4, 1)$ is a trapezoid.
- Given quadrilateral $JKLM$ with vertices $J(-4, 2)$, $K(1, 5)$, $L(4, 0)$, and $M(-1, -3)$:
 - Is it a trapezoid? Explain.
 - Is it a parallelogram? Explain.
 - Is it a rectangle? Explain.
 - Is it a rhombus? Explain.
 - Is it a square? Explain.
 - Name a point on the diagonal of $JKLM$. Explain how you know.

Lesson 14: Motion Along a Line—Search Robots Again

Classwork

Opening Exercise

- If $f(t) = (t, 2t - 1)$, find the values of $f(0)$, $f(1)$, and $f(5)$, and plot them on a coordinate plane.
- What is the image of $f(t)$?
- At what time does the graph of the line pass through the y -axis?
- When does it pass through the x -axis?
- Can you write the equation of the line you graphed in slope y -intercept form?
- How does this equation compare with the definition of $f(t)$?

Example 1

Programmers want to program a robot so that it moves at a uniform speed along a straight line segment connecting two points A and B . If $A(0, -1)$ and $B(1, 1)$, and the robot travels from A to B in $t = 1$ minute,

- Where is the robot at $t = 0$?
- Where is the robot at $t = 1$?
- Draw a picture that shows where the robot will be at $0 < t < 1$.

Exercise 1

A robot is programmed to move along a straight line path through two points A and B . It travels at a uniform speed that allows it to make the trip from $A(0, -1)$ to $B(1, 1)$ in $t = 1$ minute. Find the location, P , when

- $t = \frac{1}{4}$
- $t = 0.7$

c. $t = \frac{5}{4}$

d. $t = 2.2$

Example 2

Our robot has been reprogrammed so that it moves along the same straight line path through two points $A(0, -1)$ and $B(1, 1)$ at a uniform rate but makes the trip in 0.6 minutes instead of 1 minute.

How does this change the way we calculate the location of the robot at any time, t ?

- Find the location, P , of the robot from Example 1 if the robot were traveling at a uniform speed that allowed it to make the trip from A to B in $t = 0.6$ minutes. Is the robot's speed greater or less than the robot's speed in Example 1?
- Find the location, P , of the robot from Example 1 if the robot were traveling at a uniform speed that allowed it to make the trip from A to B in $t = 1.5$ minutes. Is the robot's speed greater or less than the robot's speed in Example 1?

Exercise 2

Two robots are moving along straight line paths in a rectangular room. Robot 1 starts at point $A(20,10)$ and travels at a constant speed to point $B(120,50)$ in two minutes. Robot 2 starts at point $C(90,10)$ and travels at a constant speed to point $D(60,70)$ in 90 seconds.

- Find the location, P , of Robot 1 after it has traveled for t minutes along its path from A to B .
- Find the location, Q , of Robot 2 after it has traveled for t minutes along its path from A to B .
- Are the robots traveling at the same speed? If not, which robot's speed is greater?
- Are the straight line paths that the robots are traveling parallel, perpendicular, or neither? Explain your answer.

Example 3

A programmer wants to program a robot so that it moves at a constant speed along a straight line segment connecting the point $A(30,60)$ to the point $B(200,100)$ over the course of a minute.

At time $t = 0$, the robot is at point A .

At time $t = 1$, the robot is at point B .

- a. Where will the robot be at time $t = \frac{1}{2}$?

- b. Where will the robot be at time $t = 0.6$?

Problem Set

1. Find the coordinates of the intersection of the medians of $\triangle ABC$ given $A(2,4)$, $B(-4,0)$, and $C(3,-1)$.
2. Given a quadrilateral with vertices $A(-1,3)$, $B(1,5)$, $C(5,1)$, and $D(3,-1)$:
 - a. Prove that quadrilateral $ABCD$ is a rectangle.
 - b. Prove that $(2,2)$ is a point on both diagonals of the quadrilateral.
3. The robot is programed to travel along a line segment at a constant speed. If P represents the robot's position at any given time t in minutes:

$$P = (240, 60) + \frac{t}{10}(100, 100),$$

- a. What was the robot's starting position?
 - b. Where did the robot stop?
 - c. How long did it take the robot to complete the entire journey?
 - d. Did the robot pass through the point $(310, 130)$ and, if so, how long into its journey did the robot reach this position?
4. Two robots are moving along straight line paths in a rectangular room. Robot 1 starts at point $A(20,10)$ and travels at a constant speed to point $B(120, 50)$ in two minutes. Robot 2 starts at point $C(90,10)$ and travels at a constant speed to point $D(60,70)$ in 90 seconds. If the robots begin their journeys at the same time, will the robots collide? Why or why not?

Lesson 15: The Distance from a Point to a Line

Classwork

Exercise 1

A robot is moving along the line $20x + 30y = 600$. A homing beacon sits at the point $(35, 40)$.

- a. Where on this line will the robot hear the loudest ping?

- b. At this point, how far will the robot be from the beacon?

Exercise 2

For the following problems, use the formula to calculate the distance between the point P and the line l .

$$d = \sqrt{\left(\frac{p + qm - bm}{1 + m^2} - p\right)^2 + \left(m\left(\frac{p + qm - bm}{1 + m^2}\right) + b - q\right)^2}$$

a. $P(0,0)$ and the line $y = 10$

b. $P(0,0)$ and the line $y = x + 10$

c. $P(0,0)$ and the line $y = x - 6$

Problem Set

- Given $\triangle ABC$ with vertices $A(3, -1)$, $B(2, 2)$, and $C(5, 1)$.
 - Find the slope of the angle bisector of $\angle ABC$.
 - Prove that the bisector of $\angle ABC$ is the perpendicular bisector of \overline{AC} .
 - Write the equation of the line containing \overline{AD} .
- Use the distance formula from today's lesson to find the distance between the point $P(-2, 1)$ and the line $y = 2x$.
- Confirm the results obtained in Problem 1 using another method.
- Find the perimeter of quadrilateral $DEBF$ shown below.

