

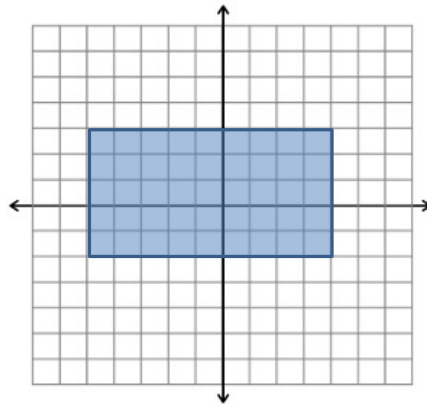
Name _____

Date _____

Lesson 2: Finding Systems of Inequalities That Describe Triangular and Rectangular Regions

Exit Ticket

Given the region shown:



- Name three points in the region.
- Identify the coordinates of the four vertices.
- Write the system of inequalities describing this region.

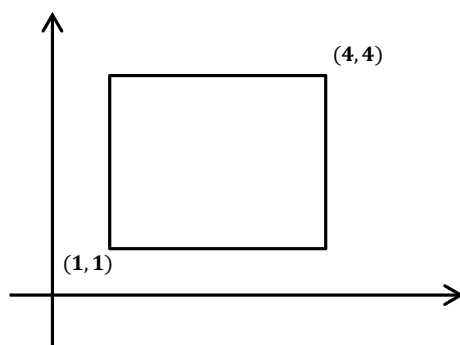
Name _____

Date _____

Lesson 3: Lines That Pass Through Regions

Exit Ticket

Consider the rectangular region:



- a. Does a line with slope $\frac{1}{2}$ passing through the origin intersect this region? If so, what are the boundary points it intersects? What is the length of the segment within the region?
- b. Does a line with slope 3 passing through the origin intersect this region? If so, what are the boundary points it intersects?

Name _____

Date _____

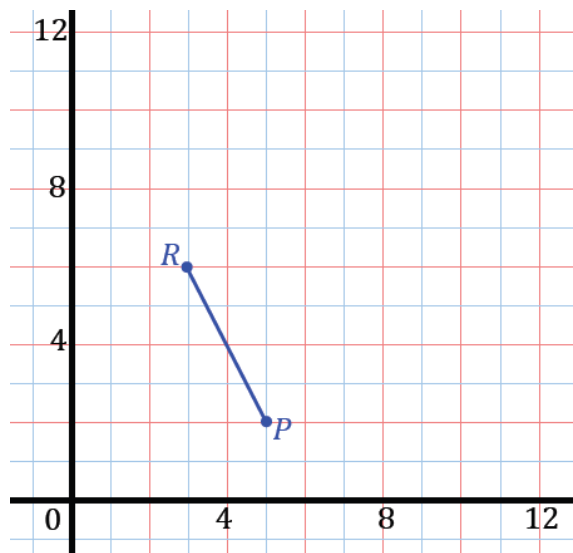
Lesson 4: Designing a Search Robot to Find a Beacon

Exit Ticket

If the line segment connecting point $P(5, 2)$ to $R(3, 6)$ is rotated 90° counterclockwise about point R :

a. Where will point P land?

b. What is the slope of the original segment, \overline{PR} ?



c. What is the slope of the rotated segment? Explain how you know.

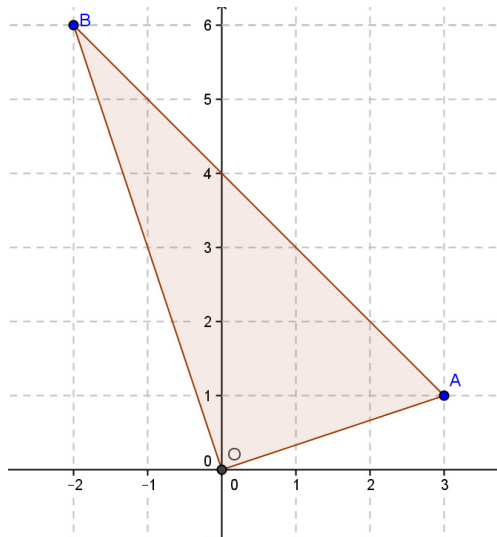
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Date _____

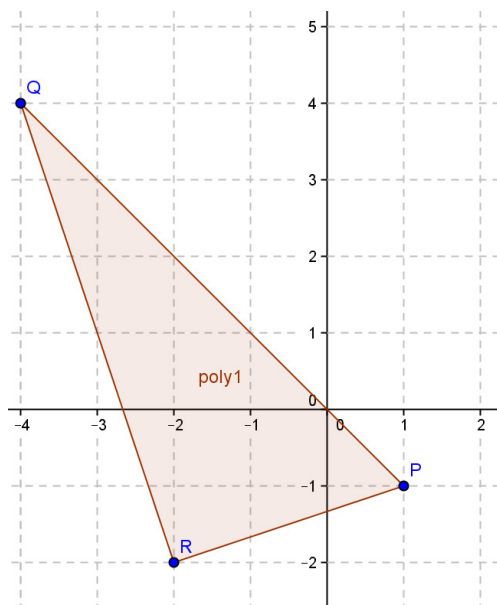
Lesson 5: Criterion for Perpendicularity

Exit Ticket

1. Given points $O(0,0)$, $A(3,1)$, and $B(-2,6)$, prove \overline{OA} is perpendicular to \overline{OB} .



2. Given points $P(1,-1)$, $Q(-4,4)$, and $R(-2,-2)$, prove \overline{PR} is perpendicular to \overline{QR} without the Pythagorean theorem.



Name _____

Date _____

Lesson 6: Segments That Meet at Right Angles

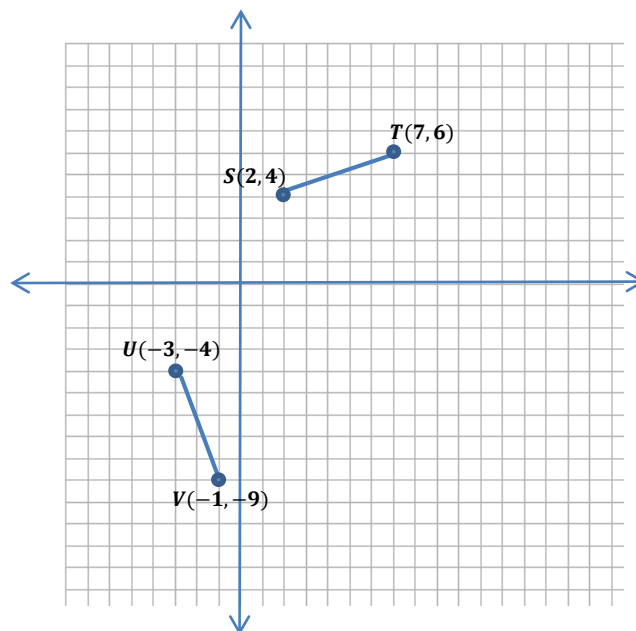
Exit Ticket

Given points $S(2, 4)$, $T(7, 6)$, $U(-3, -4)$, and $V(-1, -9)$:

- a. Translate segments \overline{ST} and \overline{UV} so that the image of each segment has an endpoint at the origin.

- b. Are the segments perpendicular? Explain.

- c. Are the lines \overleftrightarrow{ST} and \overleftrightarrow{UV} perpendicular? Explain.



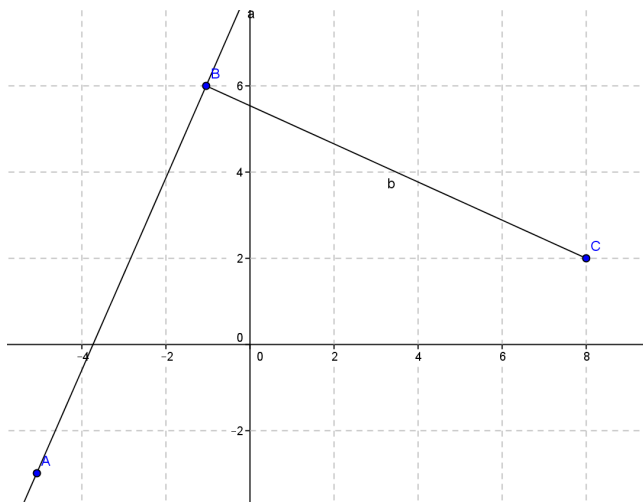
Name _____

Date _____

Lesson 7: Equations for Lines Using Normal Segments

Exit Ticket

Given $A(-5, -3)$, $B(-1, 6)$, and $C(x, y)$:



a. What are the coordinates of the translated points if B moves to the origin?

b. Write the condition for perpendicularity equation.

c. Write the equation for the normal line.

Name _____

Date _____

Lesson 8: Parallel and Perpendicular Lines

Exit Ticket

1. Are the pairs of lines parallel, perpendicular, or neither? Explain.

a. $3x + 2y = 74$ and $9x - 6y = 15$

b. $4x - 9y = 8$ and $18x + 8y = 7$

2. Write the equation of the line passing through $(-3, 4)$ and perpendicular to $-2x + 7y = -3$.

Name _____

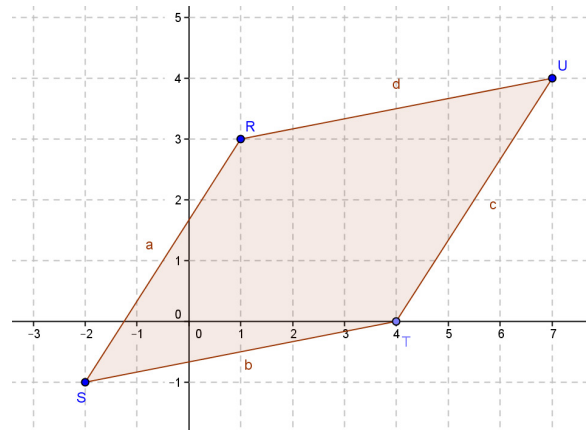
Date _____

For problems that require rounding, round answers to the nearest hundredth unless otherwise stated.

1. Given parallelogram $RSTU$ with vertices $R(1, 3)$, $S(-2, -1)$, $T(4, 0)$, and $U(7, 4)$:

- a. Find the perimeter of the parallelogram; round to the nearest hundredth.

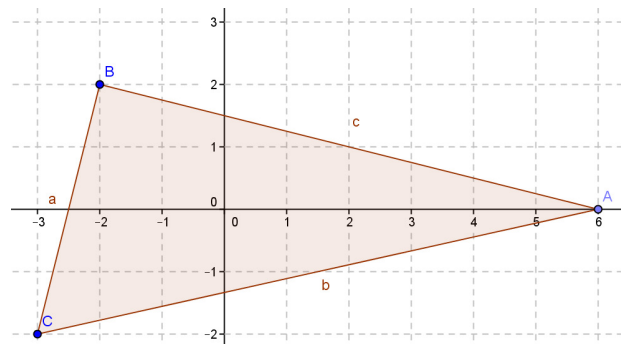
- b. Find the area of the parallelogram.



2. Given triangle ABC with vertices $A(6, 0)$, $B(-2, 2)$, and $C(-3, -2)$:

- a. Find the perimeter of the triangle, round to the nearest hundredth.

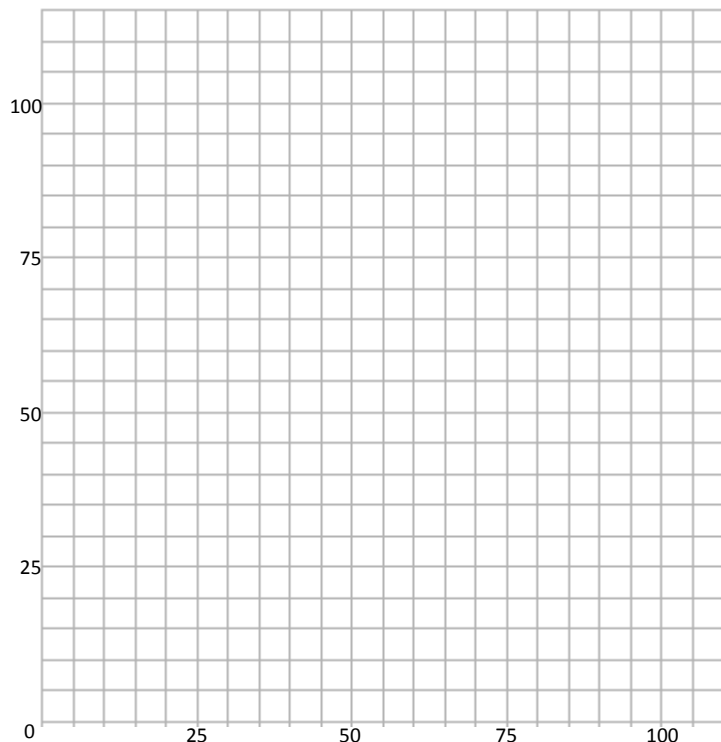
- b. Find the area of the triangle.



3. You are a member of your school's robotics team and are in charge of programming a robot that will pick up ping pong balls. The competition arena is a rectangle with height 90 feet and width 95 feet.

On graph paper, you sketch the arena as a square on the coordinate plane with sides that are parallel to the coordinate axes and with the southwest corner of the arena set at the origin. Each unit width on the paper grid corresponds to 5 feet of length of the arena. You initially set the robot to move along a straight path at a constant speed. In the sketch, the robot's position corresponds to the point $(10, 30)$ in the coordinate plane at time $t = 2$ seconds and to the point $(40, 75)$ at time $t = 8$ seconds.

- a. Sketch the arena on the graph paper below, and write a system of inequalities that describes the region in the sketch.



- b. Show that at the start, that is, at time $t = 0$, the robot was located at a point on the west wall of the arena. How many feet from the southwest corner was it?
- c. What is the speed of the robot? Round to the nearest whole number.
- d. Write down an equation for the line along which the robot moves.

- e. At some time the robot will hit a wall. Which wall will it hit? What are the coordinates of that point of impact?
- f. How far does the robot move between time $t = 0$ seconds and the time of this impact? What is the time for the impact? Round distance to the nearest hundredth and time to the nearest second.

At the time of impact, you have the robot come to a gentle halt, and then turn and head in a direction perpendicular to the wall. Just as the robot reaches the opposite wall, it gently halts, turns, and then returns to start. (We are assuming that the robot does not slow down when it hits a wall.) The robot thus completes a journey composed of three line segments forming a triangle within the arena. Sketch the path of the robot's motion.

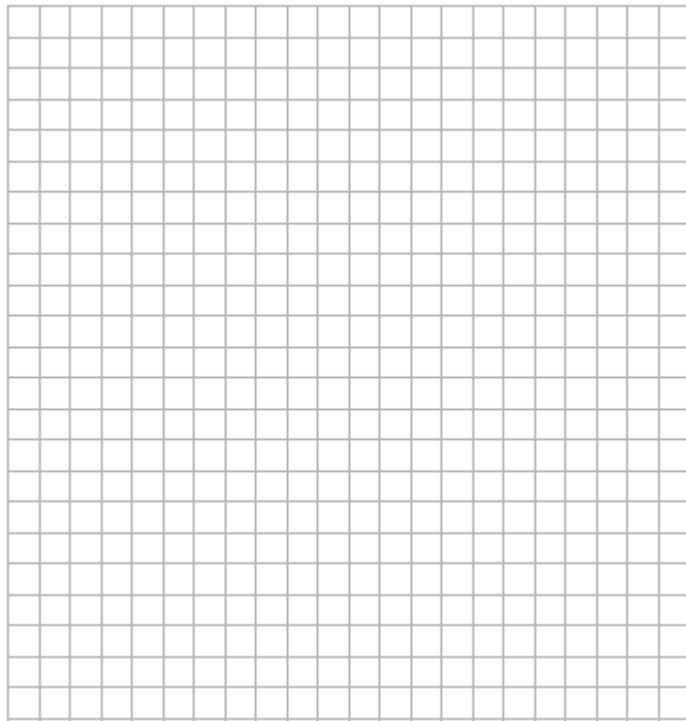
- g. What are the coordinates where it hits the east wall?
- h. What is the perimeter of that triangle rounded to the nearest hundredth?
- i. What is the area of the triangle rounded to the nearest tenth?
- j. If the count of ping-pong balls in the arena is large and the balls are spread more or less evenly across the whole arena, what approximate percentage of balls do you expect to lie within the triangle the robot traced? (Assume the robot encountered no balls along any legs of its motion.)

4. Consider the triangular region in the plane given by the triangle $(1, 6)$, $(6, -1)$, and $(1, -4)$.

a. Sketch the region and write a system of inequalities to describe the region bounded by the triangle.

b. The vertical line $x = 3$ intersects this region. What are the coordinates of the two boundary points it intersects? What is the length of the vertical segment within the region between these two boundary points?

c. The line $x = 3$ divides the region into a quadrilateral and a triangle. Find the perimeter of the quadrilateral and the area of the triangle.



5. Is triangle RST , where $R(1, 5)$, $S(5, 1)$, $T(-1, -1)$, a right triangle? If so, which angle is the right angle? Justify your answer.

6. Consider the points $A(-1, 3)$ and $B(6, 2)$ in the coordinate plane. Let $O(0, 0)$ be the origin.
- Find the coordinates of a point, C , away from the origin on the line $y = x$ that make triangle ABC a right triangle with right angle at C .
 - Find the coordinates of a point, D , on the line $y = x$ that make triangle OBD a right triangle with right angle at B .
7. Consider the quadrilateral with vertices $(-2, -1)$, $(2, 2)$, $(5, -2)$, and $(1, -5)$.
- Show that the quadrilateral is a rectangle.
 - Is the quadrilateral a square? Explain.
 - What is the area of the quadrilateral?

- d. What is the area of the region of the quadrilateral that lies to the right of the y -axis?
- e. What is the equation of the perpendicular bisector of the side of the quadrilateral that lies in the fourth quadrant?
8. Using the general formula for perpendicularity of segments with one endpoint at the origin, determine if the segments from the given points to the origin are perpendicular.
- a. $(4, 10)$, $(5, -2)$
- b. $(-7, 0)$, $(0, -4)$
- c. Using the information from part (a), are the segments through the points $(-3, -2)$, $(1, 8)$, and $(2, -4)$ perpendicular? Explain.

9. Write the equation of the line that contains the point $(-2, 7)$ and is
- a. Parallel to $x = 3$.
 - b. Perpendicular to $x = -3$.
 - c. Parallel to $y = 6x - 13$.
 - d. Perpendicular to $y = 6x - 13$.
10. Line A contains points $(p - 4, 2)$ and $(-2, 9)$. Line B contains points $(p, -1)$ and $(-1, 1)$.
- a. Find the value of p if the lines are parallel.
 - b. Find the value(s) of p if the lines are perpendicular.

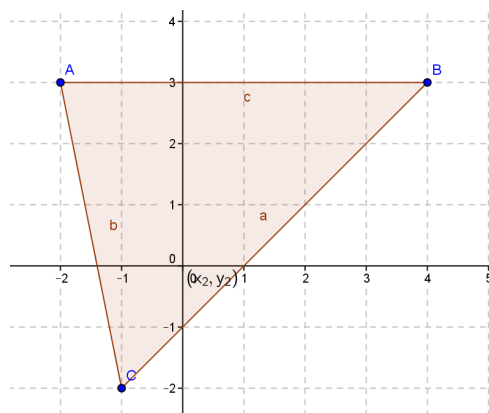
Name _____

Date _____

Lesson 9: Perimeter and Area of Triangles in the Cartesian Plane

Exit Ticket

Given the triangle below with vertices $A(-2, 3)$, $B(4, 4)$, and $C(-1, -2)$.



Azha calculated the area using $5 \cdot 6 - \frac{1}{2}(5 \cdot 1) - \frac{1}{2}(5 \cdot 5)$,

while Carson calculated the area using $\frac{1}{2}((-2) \cdot 4 + 4 \cdot (-2) + (-1) \cdot 3 - 3 \cdot 4 - 4 \cdot (-2) - (-2) \cdot (-2))$.

Explain the method each one used.

Name _____

Date _____

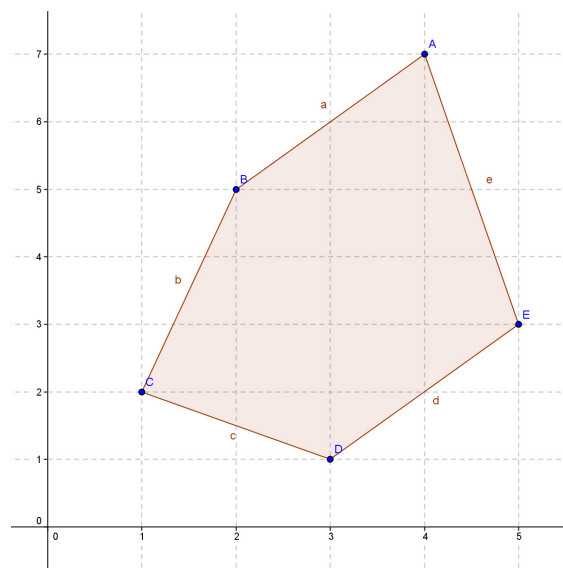
Lesson 10: Perimeter and Area of Polygonal Regions in the Cartesian Plane

Exit Ticket

Cory is using the shoelace formula to calculate the area of the pentagon shown. The pentagon has vertices $A(4, 7)$, $B(2, 5)$, $C(1, 2)$, $D(3, 1)$, and $E(5, 3)$. His calculations are below. Toya says his answer can't be correct because the area in the region is more than 2 square units. Can you identify and explain Cory's error and help him calculate the correct area?

Cory's work:

$$\frac{1}{2}(4 \cdot 5 + 2 \cdot 2 + 1 \cdot 1 + 3 \cdot 3 - 7 \cdot 2 - 5 \cdot 1 - 2 \cdot 3 - 1 \cdot 5) \\ = 2 \text{ square units}$$



Name _____

Date _____

Lesson 11: Perimeters and Areas of Polygonal Regions Defined by Systems of Inequalities

Exit Ticket

A quadrilateral region is defined by the system of inequalities below:

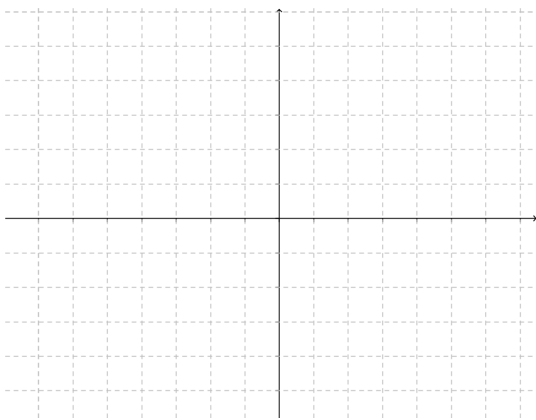
$$y \leq 5$$

$$y \geq -3$$

$$y \leq 2x + 1$$

$$y \geq 2x - 7$$

1. Sketch the region.



2. Determine the coordinates of the vertices.

3. Find the area of the quadrilateral region.

Name _____

Date _____

Lesson 12: Dividing Segments Proportionately

Exit Ticket

1. Given points $A(3, -5)$ and $B(19, -1)$, find the coordinates of point C that sits $\frac{3}{8}$ of the way along \overline{AB} , closer to A than to B .
2. Given points $A(3, -5)$ and $B(19, -1)$, find the coordinates of point C such that $\frac{CB}{AC} = \frac{1}{7}$.

Name _____

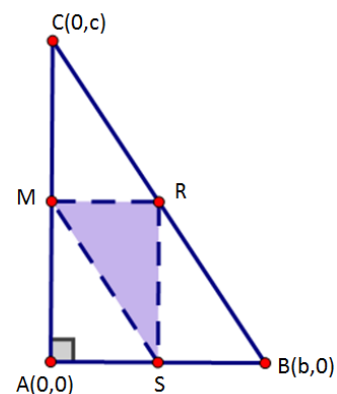
Date _____

Lesson 13: Analytic Proofs of Theorems Previously Proved by Synthetic Means

Exit Ticket

Prove that the medians of any right triangle form similar right triangles whose area is $\frac{1}{4}$ the area of the original triangle.

Prove the area of $\triangle CMR$ is $\frac{1}{4}$ the area of $\triangle CAB$.



Name _____

Date _____

Lesson 14: Motion Along a Line—Search Robots Again

Exit Ticket

Programmers want to program a robot so that it moves along a straight line segment connecting the point $A(35, 80)$ to the point $B(150, 15)$ at a uniform speed over the course of five minutes. Find the location of the robot at the following times:

a. $t = 0$

b. $t = 2$

c. $t = 3.5$

d. $t = 5$

Name _____

Date _____

Lesson 15: The Distance from a Point to a Line

Exit Ticket

1. Find the distance between the point $P(0,0)$ and the line $y = -x + 4$ using the formula from today's lesson.

2. Verify using another method.

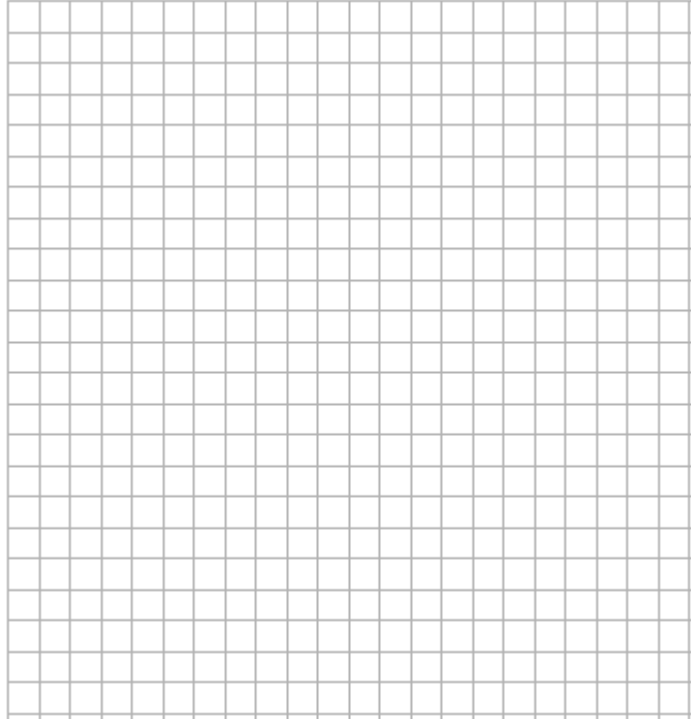
Name _____

Date _____

For problems that require rounding, round answers to the nearest hundredth.

1. A triangular region in the coordinate plane is defined by the system of inequalities $y \geq \frac{1}{2}x - 6$, $y \leq -2x + 9$, $y \leq 8x + 9$.

- a. Determine the coordinates of the vertices of the triangle.
- b. Sketch the triangular region defined by these inequalities.
- c. Is the triangle defined by the inequalities a right triangle? Explain your answer.



- d. Find the perimeter of the triangular region defined by the inequalities, round to the nearest hundredth.

- e. What is the area of this triangular region?
- f. Of the three altitudes of the triangular region defined by the inequalities, what is the length of the shortest of the three? Round to the nearest hundredth.
2. Find the point on the directed line segment from $(0,3)$ to $(6,9)$ that divides the segment in the ratio of 2:1.
3. Consider the points $A(1,4)$ and $B(8,-3)$. Suppose C and D are points on the line through A and B satisfying $\frac{AC}{CB} = \frac{1}{3}$ and $\frac{BD}{DA} = \frac{4}{3}$, respectively.
- a. Draw a sketch of the four collinear points A, B, C, D showing their relative positions to one another.

b. Find the coordinates of the point C .

c. Find the coordinates of the point D .

4. Two robots are left in a robotics competition. Robot A is programmed to move about the coordinate plane at a constant speed so that, at time t seconds, its position in the plane is given by

$$(0, 10) + \frac{t}{8}(60, 80).$$

Robot B is also programmed to move about the coordinate plane at a constant speed. Its position in the plane at time t seconds is given by

$$(70, 0) - \frac{t}{10}(70, -70).$$

a. What was each robot's starting position?

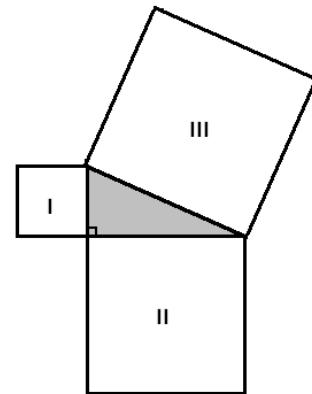
b. Where did each robot stop?

- c. What is the equation of the path of robot A?
- d. What is the equation of the path of robot B?
- e. What is the speed of robot A? (Assume coordinates in the plane are given in units of meters. Give the speed in units of meters per second.)
- f. Do the two robots ever pass through the same point in the plane? Explain. If they do, do they pass through that common point at the same time? Explain.
- g. What is the closest distance robot B will ever be to the origin? Round to the nearest hundredth.

- h. At time $t = 10$ seconds, robot A will instantaneously turn 90 degrees to the left and travel at the same constant speed it was previously traveling. What will be its coordinates in another 10 seconds' time?
5. $GDAY$ is a rhombus. If point G has coordinates $(2, 6)$ and A has coordinates $(8, 10)$, what is the equation of the line that contains the diagonal \overline{DY} of the rhombus?
6. a. A triangle has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$. Let M be the midpoint of \overline{AC} and N the midpoint of \overline{BC} . Find a general expression for the slope of \overline{MN} . What segment of the triangle has the same slope as \overline{MN} ?

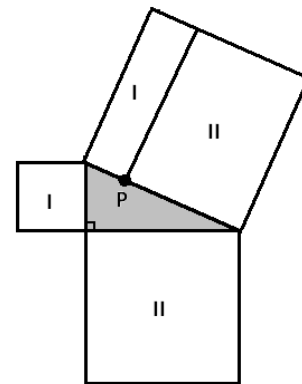
- b. A triangle has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$. Let P be a point on \overline{AC} with $AP = \frac{5}{8}AC$, and let Q be a point on \overline{BC} with $BQ = \frac{5}{8}BC$. Find a general expression for the slope of \overline{PQ} . What segment of the triangle has the same slope as \overline{PQ} ?
- c. A quadrilateral has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$, and $D(d_1, d_2)$. Let R , S , T , and U be the midpoints of the sides \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} , respectively. Demonstrate that \overline{RS} is parallel to \overline{TU} . Is \overline{ST} parallel to \overline{UR} ? Explain.

7. The Pythagorean theorem states that if three squares are drawn on the sides of a right triangle, then the area of the largest square equals the sum of the areas of the two remaining squares.



$$\text{area III} = \text{area I} + \text{area II}$$

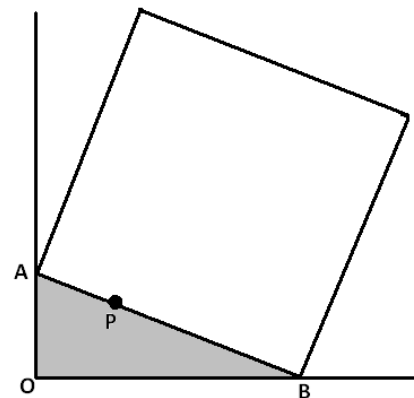
There must be a point P along the hypotenuse of the right triangle at which the large square is divided into two rectangles as shown, each with area matching the area of one of the smaller squares.



Consider a right triangle AOB situated on the coordinate plane with vertex A on the positive y -axis, O the origin, and vertex B on the positive x -axis.

Suppose A has coordinates $(0, a)$, B has coordinates $(b, 0)$, and the length of the hypotenuse \overline{AB} is c .

- a. Find the coordinates of a point P on the line segment \overline{AB} such that \overline{OP} is perpendicular to \overline{AB} .



- b. Show that for this point P we have $\frac{AP}{PB} = \frac{a^2}{b^2}$.
- c. Show that if we draw from P a line perpendicular to \overline{AB} , then that line divides the square with \overline{AB} as one of its sides into two rectangles, one of area a^2 and one of area b^2 .