



Topic A:

The Story of Trigonometry and Its Contexts

F-IF.C.7e, F-TF.A.1, F-TF.A.2

Focus Standard:	F-IF.C.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. [★] e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
	F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
	F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measure of angles traversed counterclockwise around the unit circle.
Instructional Days:	10	
Lesson 1:	Ferris Wheels—Tracking the Height of a Passenger Car (E) ¹	
Lesson 2:	The Height and Co-Height Functions of a Ferris Wheel (E)	
Lesson 3:	The Motion of the Moon, Sun, and Stars—Motivating Mathematics (S)	
Lesson 4:	From Circle-ometry to Trigonometry (S)	
Lesson 5:	Extending the Domain of Sine and Cosine to All Real Numbers (S)	
Lesson 6:	Why Call It Tangent? (S)	
Lesson 7:	Secant and the Co-Functions (S)	
Lesson 8:	Graphing the Sine and Cosine Functions (E)	
Lesson 9:	Awkward! Who Chose the Number 360, Anyway? (S)	
Lesson 10:	Basic Trigonometric Identities from Graphs (E)	

In Topic A, students develop an understanding of the six basic trigonometric functions as functions of the amount of rotation of a point on the unit circle, and then translate that understanding to the trigonometric functions as functions on the real number line. In Lessons 1 and 2, a Ferris wheel provides a familiar context

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

for the introduction of periodic functions that will lead to the sine and cosine functions in Lessons 4 and 5. Lesson 1 is an exploratory lesson in which students model the circular motion of a Ferris wheel using a paper plate. The goal is to study the vertical component of the circular motion with respect to the degrees of rotation of the wheel from the initial position. We temporarily describe this function as the “height function” of a passenger car on the Ferris wheel, and students produce a graph of the height function from their model. In this first lesson, students begin to understand the periodicity of the height function as the Ferris wheel completes multiple rotations (MP.7).

Lesson 2 introduces the “co-height function,” which describes the horizontal component of the circular motion of the Ferris wheel. The students again model the position of a car on a rotating Ferris wheel using a paper plate, this time with emphasis on the horizontal motion of the car. In the first lesson, heights were measured from the “ground” to the passenger car of the Ferris wheel, so that the graph of the height function was contained within the first quadrant of the Cartesian plane. In this second lesson, we change our frame of reference so that the values of the height and co-height functions oscillate between $-r$ and r , where r is the radius of the wheel, inching the height and co-height functions toward the sine and cosine functions. The goal of these first two lessons is to provide a familiar context for circular motion so that students can begin to see how the horizontal and vertical components of the position of a point rotating around a circle can be described by periodic functions of the amount of rotation. We will refer to this context as needed throughout the module.

Lesson 3 provides historical background on the development of the sine and cosine functions in India around A.D. 500. In this lesson, students generate part of a sine table and use it to calculate the positions of the sun in the sky, assuming the historical model of the sun following a circular orbit around Earth. This lesson provides a second example of circular motion that can be modeled using the sine and cosine functions. In this lesson, the link is made between the assumed circular motion of stars and the sun and the periodic sine and cosine functions, and that link is formalized in Lesson 4.

Lesson 4 draws connections between the height function of a Ferris wheel and the sine and cosine functions used in triangle trigonometry in Geometry. This lesson extends the domain of the sine and cosine functions from the restricted domain ($0^\circ, 90^\circ$) of angle measures of acute angles in triangles to the interval ($0^\circ, 360^\circ$). Abstracting the sine and cosine from the height and co-height functions of the Ferris wheel allows students to practice MP.2.

In fully developing **F-TF.A.2** on extending the trigonometric functions to the entire real line in Lesson 5, students need to come to know enough values of these functions to generate graphs of these functions and discern structure and properties about them (in much the same way that students were first introduced to exponential functions by studying their values at integer inputs). The most important values to learn, of course, are the values of sine and cosine functions of the most commonly used reference points: the sine and cosine of degree measures that are multiples of 30° and 45° . This knowledge, in turn, serves as concrete examples for learning standard **F-TF.A.1**.

Lesson 6 and 7 introduce the tangent and secant functions through their geometric descriptions on a circle, and link those geometric descriptions to the appropriate ratios of sine and cosine. The remaining trigonometric functions cotangent and cosecant are also introduced.

In Lesson 8, students construct a graph of the sine and cosine functions as functions on the real line by measuring the horizontal and vertical components of a point on the unit circle, breaking a piece of spaghetti to the appropriate length, and gluing it to the graph. Physically creating the graphs using direct measurement ties together the definition of $\sin(\theta)$ as the y -coordinate of the point on the unit circle that has been rotated

θ about the origin from the point $(1,0)$ and the value of the periodic function $f(\theta) = \sin(\theta)$. Working now in radians, we return to finding the values of the sine and cosine functions for multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$, which aligns with **F-TF.A.3**.

As students transition to using radian measures in Lesson 9, these reference points and their trigonometric values help students to make sense of radian measure. The goal of this work, which began in the Geometry course, is for students to fluently and automatically derive/recall these values quickly in the Pre-Calculus and Advanced Topics course, thereby satisfying the expectation of **F-TF.A.3**. We justify the switch to radians by drawing the graph of $y = \sin(x)$ with the same scale on the horizontal and vertical axes, which is difficult to draw. This somewhat artificial task serves many different purposes; it provides justification for the use of radian measures without referring directly to ideas of calculus, it foreshadows the lessons to come in Topic B on transforming the graph of the sine function, and it allows students to look for patterns. Students practice MP.7 when they discover the effects of changing the parameters on the graph, and they practice MP.8 when they repeatedly draw graphs of sinusoidal functions to notice the patterns. Drawing on their experience with graphing parabolas $y = kx^2$, students experiment with graphing calculators to produce graphs of $y = \sin(kx)$ until they find that when $k \approx 57$ (or $k = \frac{180}{\pi}$), the line $y = x$ is tangent to $y = \sin(kx)$ at the origin. Although we define the sine and cosine functions explicitly as functions of the amount of rotation of the initial ray comprised of the non-negative part of the x -axis, at the end of Lesson 9 the students see that the measure of an angle θ in radians is the arc subtended by the angle as specified by **F-TF.A.1**. Radian measure is used exclusively through the remaining lessons in the module.

The topic culminates with Lesson 10, which incorporates such identities as $\sin(\pi - x) = \sin(x)$ and $\cos(2\pi - x) = \cos(x)$ for all real numbers x into an introduction to trigonometric identities that will be studied further in Topic B. In this lesson, students analyze the graphs of the sine and cosine function and note some basic properties that are apparent from the graphs and from the unit circle, such as the periodicity of sine and cosine, the even and odd properties of the functions, and the fact that the graph of the cosine function is a horizontal shift of the graph of the sine function. Students also note the intercepts and end behavior of these graphs.