



Topic B:

Understanding Trigonometric Functions and Putting Them to Use

F-IF.C.7e, F-TF.B.5, F-TF.C.8, S-ID.B.6a

Focus Standard:	F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
	F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
	F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
	S-ID.B.6a	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i>
Instructional Days: 7		
	Lesson 11:	Transforming the Graph of the Sine Function (E) ¹
	Lesson 12:	Ferris Wheels—Using Trigonometric Functions to Model Cyclical Behavior (E)
	Lesson 13:	Tides, Sound Waves, and Stock Markets (P)
	Lesson 14:	Graphing the Tangent Function (E)
	Lesson 15:	What Is a Trigonometric Identity? (P)
	Lesson 16:	Proving Trigonometric Identities (P)
	Lesson 17:	Trigonometric Identity Proofs (P)

In Topic A, the students developed the ideas behind the six basic trigonometric functions, focusing primarily on the sine function. In Topic B, students will use trigonometric functions to model periodic behavior. We end the module with the study of trigonometric identities and how to prove them.

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Lesson 11 continues the idea started in Lesson 9 in which students graphed $y = \sin(kx)$ for different values of k . In Lesson 11, teams of students work to understand the effect of changing the parameters A , ω , h , and k in the graph of the function $y = A(\sin(\omega(x - h))) + k$, so that in Lesson 12 students can fit sinusoidal functions to given scenarios, which aligns with **F-IF.C.7e** and **F-TF.B.5**. While Lesson 12 requires that students find a formula that precisely models periodic motion in a given scenario, Lesson 13 is distinguished by non-exact modeling, as in **S-ID.B.6a**. In Lesson 13, students analyze given real-world data and fit the data with an appropriate sinusoidal function, providing authentic practice with MP.3 and MP.4 as they debate about appropriate choices of functions and parameters.

In Lesson 14, we return to the idea of graphing functions on the real line and produce graphs of $y = \tan(x)$. Students work in groups to produce the graph of one branch of the tangent function by plotting points on a specified interval. The individual graphs are compiled into one classroom graph to emphasize the periodicity and basic properties of the tangent function.

To wrap up the module, students revisit the idea of mathematical proof in Lessons 15–17. Lesson 15 aligns with standard **F-TF.C.8**, proving the Pythagorean identity. In Lesson 17, students discover the formula for $\sin(\alpha + \beta)$ using MP.8, in alignment with standard **F-TF.B.9(+)**, but teachers may choose to present the optional rigorous proof of this formula that is provided in the lesson. Standard **F-TF.B.9(+)** is included because it logically coheres with the rest of the content in the module. Throughout Lessons 15, 16, and 17, the emphasis is on the proper statement of a trigonometric identity as the pairing of a statement that two functions are equivalent on a given domain and an identification of that domain. For example, the identity “ $\sin^2(\theta) + \cos^2(\theta) = 1$ for all real numbers θ ” is a statement that the two functions $f_1(\theta) = \sin^2(\theta) + \cos^2(\theta)$ and $f_2(\theta) = 1$ have the same value for every real number θ . As students revisit the idea of proof in these lessons, they are prompted to follow the steps of writing a valid proof:

1. Define the variables. For example, “Let θ be any real number.”
2. Establish the identity by starting with the expression on one side of the equation and transforming it into the expression on the other side through a sequence of algebraic steps using rules of logic, algebra, and previously established identities. For example:

$$\begin{aligned}\cos(2\theta) &= \cos(\theta + \theta) \\ &= \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta) \\ &= \cos^2(\theta) - \sin^2(\theta).\end{aligned}$$

3. Conclude the proof by stating the identity in its entirety. For example, “Then, for any real number θ , $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$.”