Lesson 5: Extending the Domain of Sine and Cosine to All Real Numbers

Classwork

Opening Exercises

* 1. Suppose that a group of coworkers pool their money, buying a single lottery ticket every day with the understanding that if any ticket was a winning ticket, the group would split the winnings evenly, and they would donate any left over money to the local high school. Using this strategy, the group won . How much money was donated to the school?
  2. What if the winning ticket was worth ? Using the same plan as in part (a), how much money would be donated to the school?
  3. What if the winning ticket was worth ? Using the same plan as in part (a), how much money would be donated to the school?

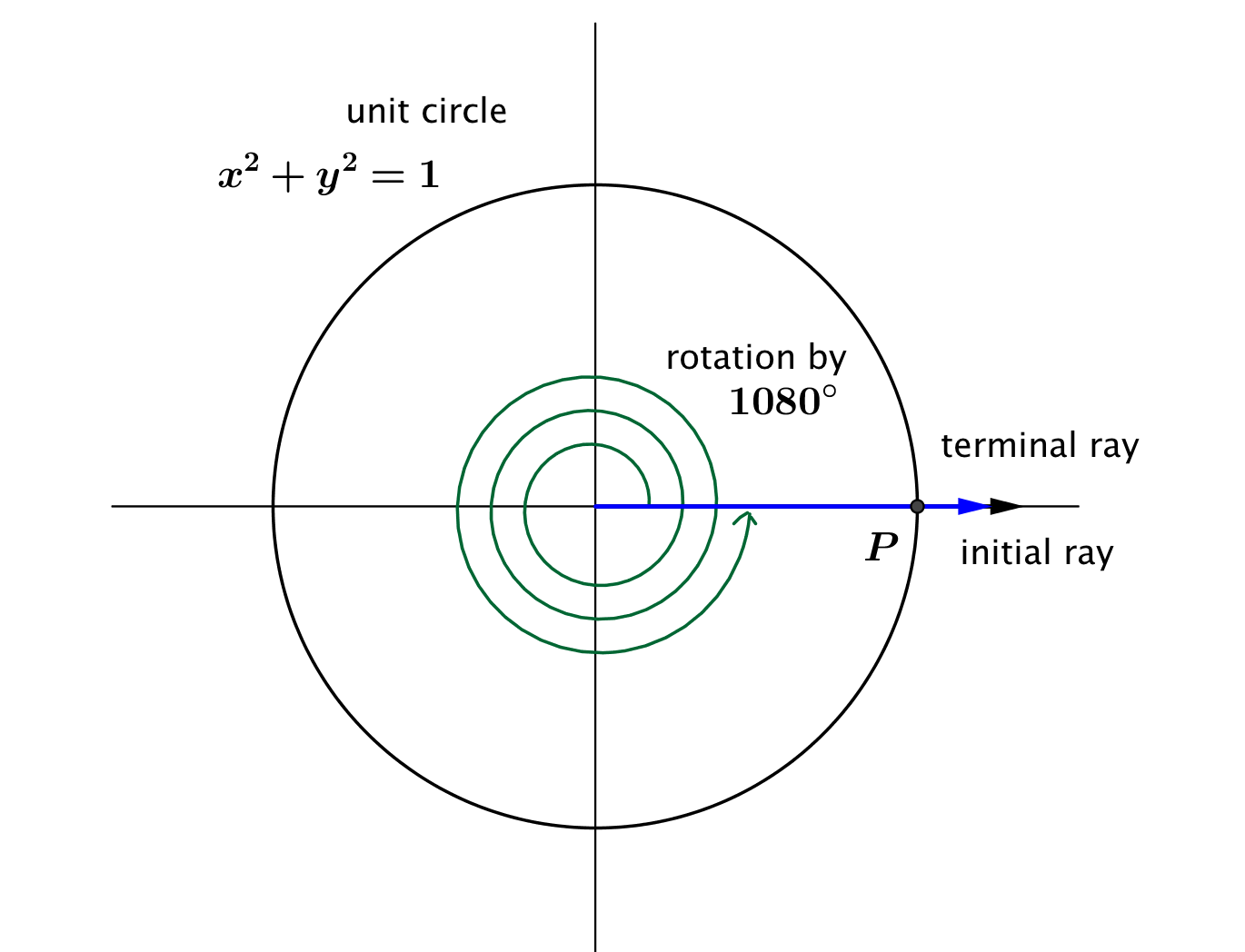
Exercises 1–5

1. Find and Identify the measure of the reference angle.
2. Find and Identify the measure of the reference angle.
3. Find and Identify the measure of the reference angle.
4. Find and Identify the measure of the reference angle.
5. Find and Identify the measure of the reference angle.

Exercises 6–10

1. Find and Identify the measure of the reference angle.
2. Find and Identify the measure of the reference angle.
3. Find and Identify the measure of the reference angle.
4. Find and Identify the measure of the reference angle.
5. Find and Identify the measure of the reference angle.

Discussion

**Case 1:** What about the values of the sine and cosine function of other amounts of rotation that produce a terminal ray along the positive -axis, such as ?

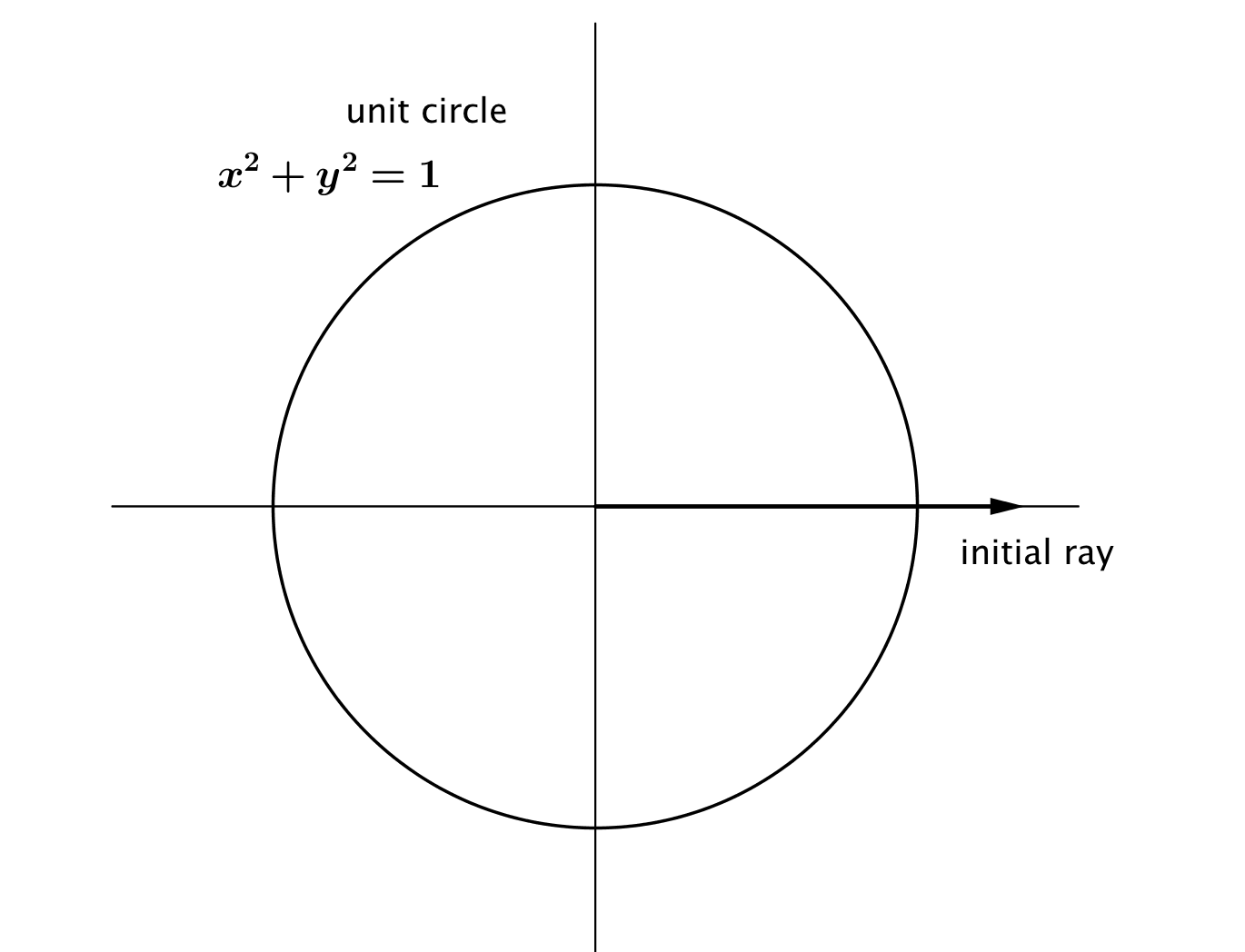
Our definition of a reference angle is the angle formed by the terminal ray and the -axis, but our terminal ray lies along the -axis so the terminal ray and the -axis form a zero angle.

How would we assign values to and ?

What if we rotated around , which is turns? What are and ?

State a generalization of these results:

If , for some integer then , and .

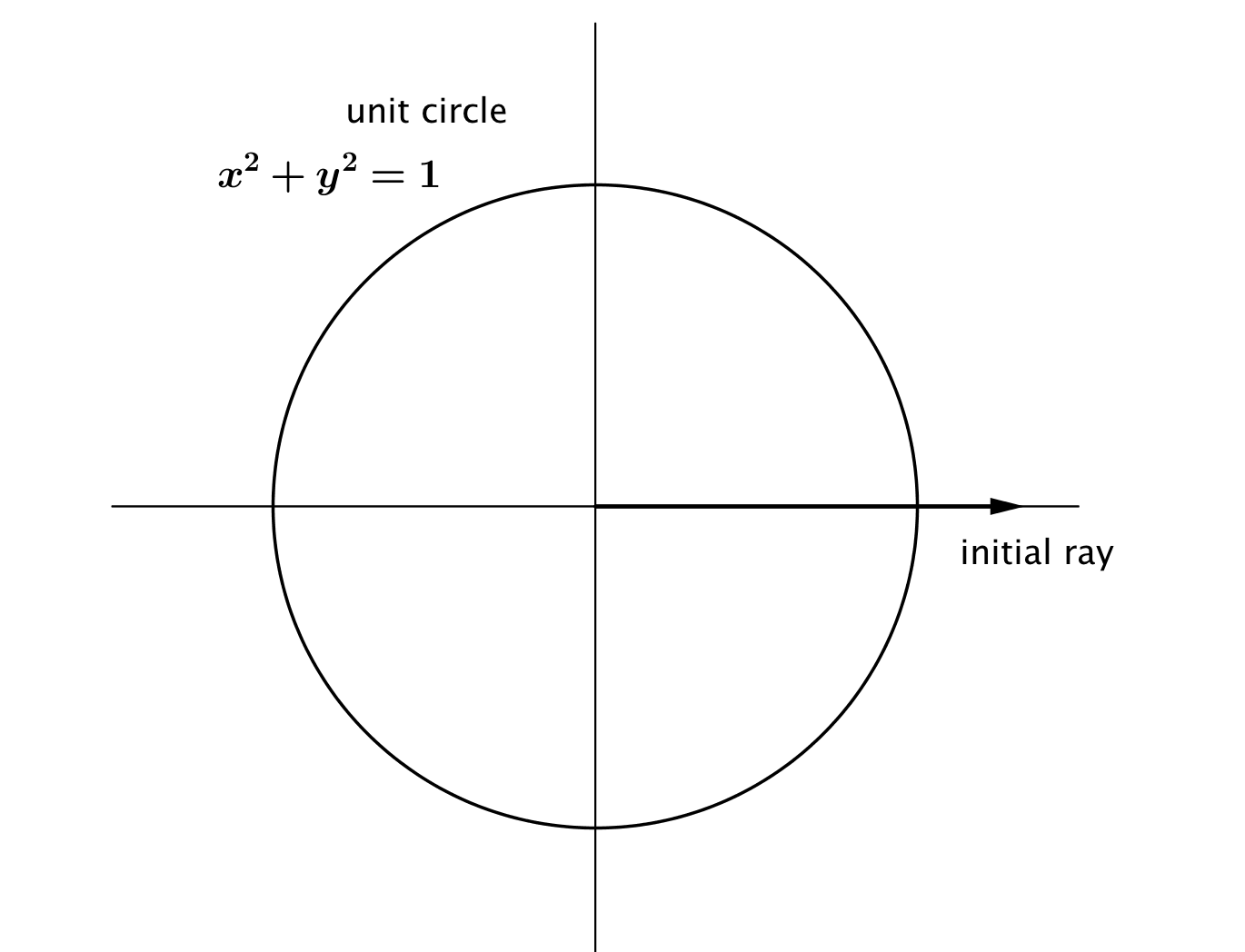
**Case 2:** What about the values of the sine and cosine function of other amounts of rotation that produce a terminal ray along the negative -axis, such as ?

How would we assign values to and ?

What are the values of and ? How do you know?

State a generalization of these results:

If , for some integer then , and .

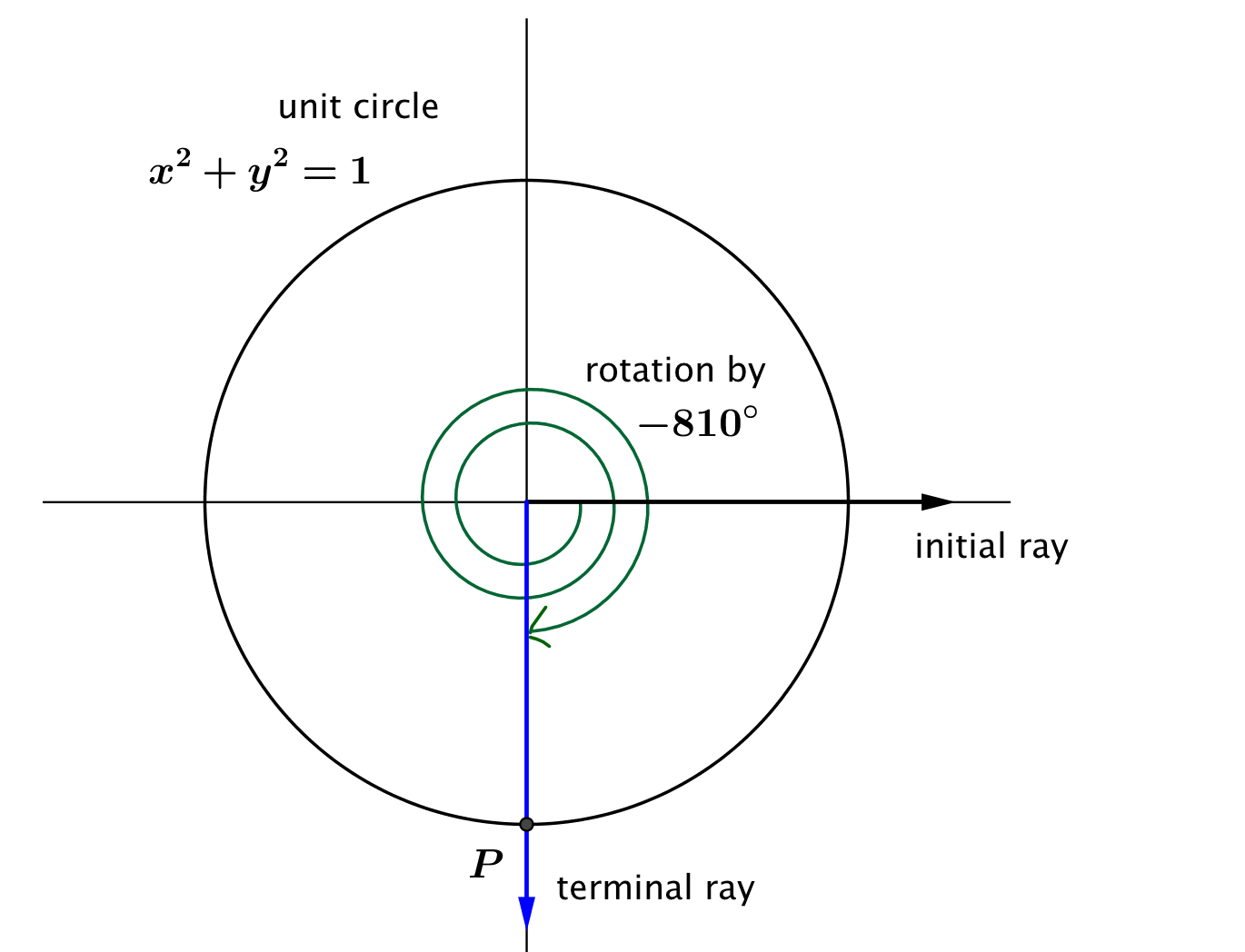
**Case 3:** What about the values of the sine and cosine function for rotations that are more than a number of full turns, such as ?How would we assign values to and

Can we generalize to any rotation that produces a terminal ray along the positive -axis?

State a generalization of these results:

*If , for some integer then , and .*

**Case 4:** What about the values of the sine and cosine function for rotations whose terminal ray lies along the negative -axis, such as ?

****How would we assign values to and ?

Can we generalize to any rotation that produces a terminal ray along the negative -axis?

State a generalization of these results:

*If , for some integer then , and .*

Discussion

Let be any real number. In the Cartesian plane, rotate the initial ray by degrees about the origin. Intersect the resulting terminal ray with the unit circle to get a point in the coordinate plane. The value of is and the value of is .

Lesson Summary

In this lesson we formalized the definition of the sine and cosine functions of a number of degrees of rotation, . We rotate the initial ray made from the positive -axis through degrees, going counterclockwise if and clockwise if . The point is defined by the intersection of the terminal ray and the unit circle.

* The value of is the -coordinate of .
* The value of is the -coordinate of .
* The sine and cosine functions have domain of all real numbers and range .

Problem Set

1. Fill in the chart; write the quadrant where the terminal ray is located after rotation by the measures of the reference angles, and the values of the sine and cosine functions for the indicated rotation numbers.

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| Number of degrees of rotation, | Quadrant | Measure of Reference  Angle |  |  |
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1. Using geometry, Jennifer correctly calculated that . Based on this information, fill in the chart:

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| --- | --- | --- | --- | --- |
| Number of degrees of rotation, | Quadrant | Measure of Reference Angle |  |  |
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1. Suppose represents a quantity in degrees, and that . List the first six possible positive values that can take.
2. Suppose represents a quantity in degrees, and that . List six possible negative values that can take.
3. Suppose represents a quantity in degrees. Is it possible that and ?
4. Jane says that since the reference angle for a rotation through has measure , then and . Explain why she is or is not correct.
5. Doug says that since the reference angle for a rotation through has measure , then and . Explain why he is or is not correct.