Lesson 37: A Surprising Boost from Geometry

Classwork

Opening Exercise

Solve each equation for x.

a. $x - 1 = 0$

b. $x + 1 = 0$

c. $x^2 - 1 = 0$

d. $x^2 + 1 = 0$

Example 1: Addition with Complex Numbers

Compute $(3 + 4i) + (7 - 20i)$.

© 2014 Common Core, Inc. Some rights reserved. commoncore.org
This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

Example 2: Subtraction with Complex Numbers

Compute $(3 + 4i) - (7 - 20i)$.

Example 3: Multiplication with Complex Numbers

Compute $(1 + 2i)(1 - 2i)$.

Example 4: Multiplication with Complex Numbers

Verify that $-1 + 2i$ and $-1 - 2i$ are solutions to $x^2 + 2x + 5 = 0$.

© 2014 Common Core, Inc. Some rights reserved. commoncore.org
This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

Lesson Summary

Multiplying by i rotates every complex number in the complex plane by 90° about the origin.

Every complex number is in the form $a + bi$, where a is the real part and b is the imaginary part of the number. Real numbers are also complex numbers; the real number a can be written as the complex number $a + 0i$.

Adding two complex numbers is analogous to combining like terms in a polynomial expression.

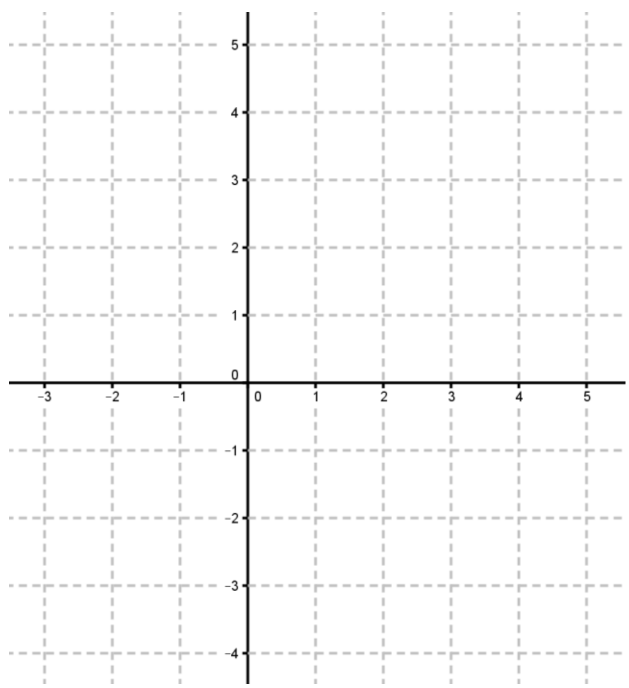
Multiplying two complex numbers is like multiplying two binomials, except one can use $i^2 = -1$ to further write the expression in simpler form.

Complex numbers satisfy the associative, commutative, and distributive properties.

Complex numbers can now allow us to find solutions to equations that previously had no real number solutions.

Problem Set

1. Locate the point on the complex plane corresponding to the complex number given in parts (a)–(h). On one set of axes, label each point by its identifying letter. For example, the point corresponding to $5 + 2i$ should be labeled "a."
   a. $5 + 2i$
   b. $3 - 2i$
   c. $-2 - 4i$
   d. $-i$
   e. $\frac{1}{2} + i$
   f. $\sqrt{2} - 3i$
   g. 0
   h. $-\frac{3}{2} + \frac{\sqrt{3}}{2}i$



2. Express each of the following in $a + bi$ form.
   a. $(13 + 4i) + (7 + 5i)$
   b. $(5 - i) - 2(1 - 3i)$
   c. $\left((5 - i) - 2(1 - 3i)\right)^2$
   d. $(3 - i)(4 + 7i)$
   e. $(3 - i)(4 + 7i) - \left((5 - i) - 2(1 - 3i)\right)$

© 2014 Common Core, Inc. Some rights reserved. commoncore.org
CC BY-NC-SA This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

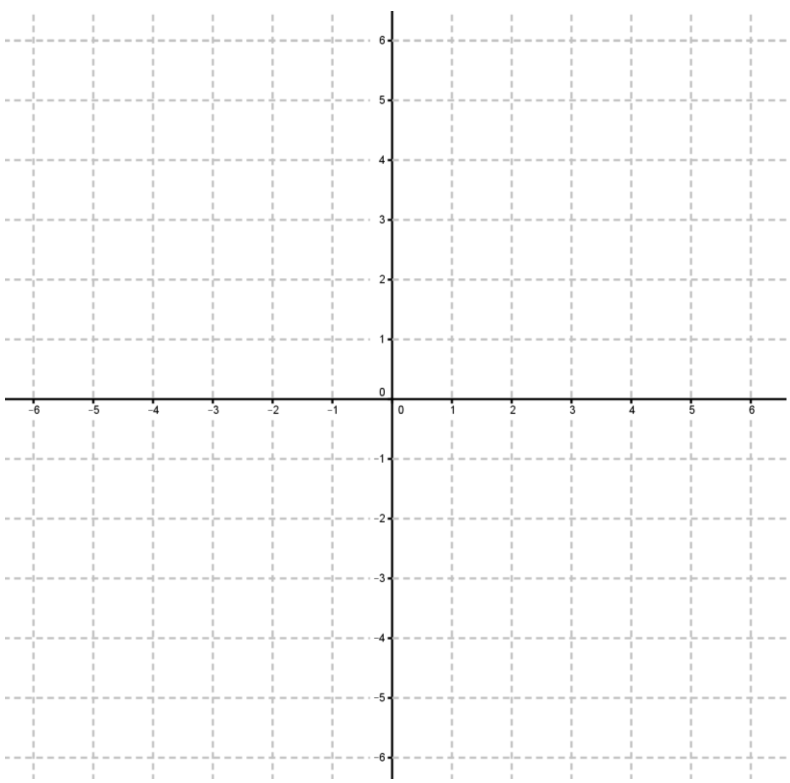
3. Express each of the following in a + bi form.
   a. $(2+5i)+(4+3i)$
   b. $(-1+2i)-(4-3i)$
   c. $(4+i)+(2-i)-(1-i)$
   d. $(5+3i)(3+5i)$
   e. $-i(2-i)(5+6i)$
   f. $(1+i)(2-3i)+3i(1-i)-i$

4. Find the real values of x and y in each of the following equations using the fact that if $a+bi=c+di$, then $a=c$ and $b=d$.
   a. $5x+3yi=20+9i$
   b. $2(5x+9)=(10-3y)i$
   c. $3(7-2x)-5(4y-3)i=x-2(1+y)i$

5. Since $i^2=-1$, we see that

$$i^3=i^2\cdot i=-1\cdot i=-i$$
$$i^4=i^2\cdot i^2=-1\cdot -1=1.$$

Plot i, i^2, i^3, and i^4 on the complex plane and describe how multiplication by each rotates points in the complex plane.



© 2014 Common Core, Inc. Some rights reserved. commoncore.org
This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

6. Express each of the following in $a + bi$ form.
   a. i^5
   b. i^6
   c. i^7
   d. i^8
   e. i^{102}

7. Express each of the following in $a + bi$ form.
   a. $(1 + \mathrm{i})^2$
   b. $(1 + \mathrm{i})^4$
   c. $(1 + \mathrm{i})^6$

8. Evaluate $x^2 - 6x$ when $x = 3 - i$.

9. Evaluate $4x^2 - 12x$ when $x = \frac{3}{2} - \frac{i}{2}$.

10. Show by substitution that $\frac{5 - i\sqrt{5}}{5}$ is a solution to $5x^2 - 10x + 6 = 0$.

11. a. Evaluate the four products below.

    Evaluate $\sqrt{9} \cdot \sqrt{4}$.

    Evaluate $\sqrt{9} \cdot \sqrt{-4}$.

    Evaluate $\sqrt{-9} \cdot \sqrt{4}$.

    Evaluate $\sqrt{-9} \cdot \sqrt{-4}$.

    b. Suppose a and b are positive real numbers. Determine whether the following quantities are equal or not equal.

    $\sqrt{a} \cdot \sqrt{b}$ and $\sqrt{-a} \cdot \sqrt{-b}$

    $\sqrt{-a} \cdot \sqrt{b}$ and $\sqrt{a} \cdot \sqrt{-b}$

© 2014 Common Core, Inc. Some rights reserved. commoncore.org
This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.