Lesson 36: Overcoming a Third Obstacle to Factoring—What If There Are No Real Number Solutions?

Classwork

Opening Exercise

Find all solutions to each of the systems of equations below using any method.

$2x - 4y = -1$ $3x - 6y = 4$	$y = x^2 - 2$ $y = 2x - 5$	$x^2 + y^2 = 1$ $x^2 + y^2 = 4$

Exercises 1–4

1. Are there any real number solutions to the system $\begin{cases} y = 4 \\ x^2 + y^2 = 2 \end{cases}$? Support your findings both analytically and graphically.

2. Does the line $y = x$ intersect the parabola $y = -x^2$? If so, how many times, and where? Draw graphs on the same set of axes.

3. Does the line $y = -x$ intersect the circle $x^2 + y^2 = 1$? If so, how many times, and where? Draw graphs on the same set of axes.

4. Does the line $y = 5$ intersect the parabola $y = 4 - x^2$? Why or why not? Draw the graphs on the same set of axes.

Lesson Summary

An equation or a system of equations may have one or more solutions in the real numbers, or it may have no real number solution.

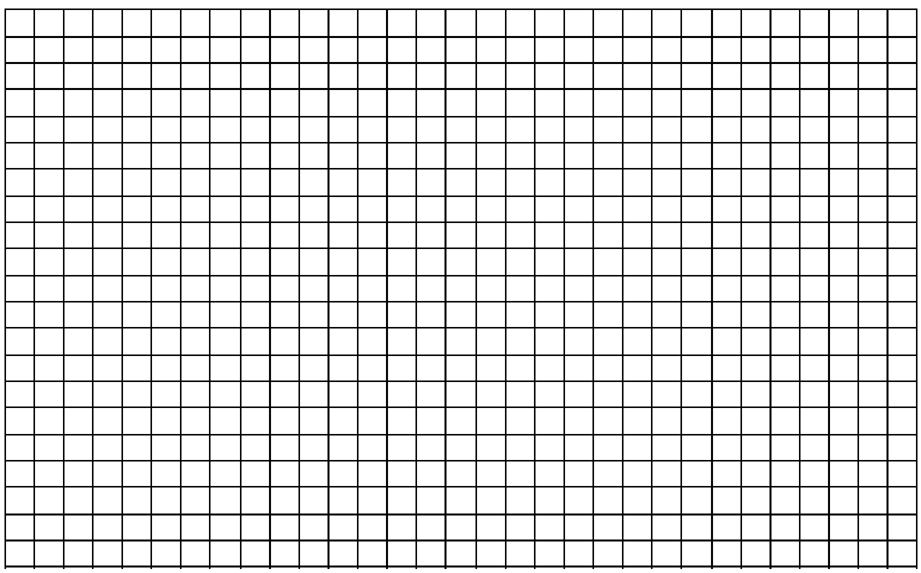
Two graphs that do not intersect in the real plane describe a system of two equations without a real solution. If a system of two equations does not have a real solution, the graphs of the two equations do not intersect in the real plane.

A quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, that has no real solution indicates that the graph of $y = ax^2 + bx + c$ does not intersect the x-axis.

Problem Set

1. For each part, solve the system of linear equations, or show that no real solution exists. Graphically support your answer.

   a. $$\begin{aligned} 4x + 2y &= 9 \\ x + y &= 3 \end{aligned}$$



b. $$\begin{aligned} 2x - 8y &= 9 \\ 3x - 12y &= 0 \end{aligned}$$

2. Solve the following system of equations, or show that no real solution exists. Graphically confirm your answer.

$$3x^2 + 3y^2 = 6$$
$$x - y = 3$$

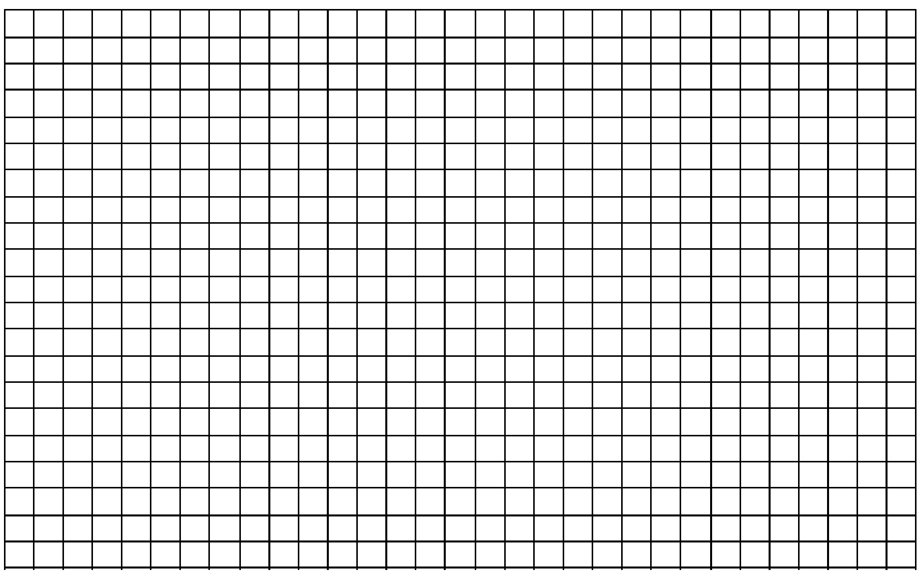
3. Find the value of k so that the graph of the following system of equations has no solution.

$$3x - 2y - 12 = 0$$
$$kx + 6y - 10 = 0$$

4. Offer a geometric explanation to why the equation $x^2 - 6x + 10 = 0$ has no real solutions.

5. Without his pencil or calculator, Joey knows that $2x^3 + 3x^2 - 1 = 0$ has at least one real solution. How does he know?

6. The graph of the quadratic equation $y = x^2 + 1$ has no x-intercepts. However, Gia claims that when the graph of $y = x^2 + 1$ is translated by a distance of 1 in a certain direction, the new (translated) graph would have exactly one x-intercept. Further, if $y = x^2 + 1$ is translated by a distance greater than 1 in the same direction, the new (translated) graph would have exactly two x-intercepts. Support or refute Gia's claim. If you agree with her, in which direction did she translate the original graph? Draw graphs to illustrate.



7. In the previous problem, we mentioned that the graph of $y = x^2 + 1$ has no x-intercepts. Suppose that $y = x^2 + 1$ is one of two equations in a system of equations and that the other equation is a line. Give an example of a linear equation such that this system has exactly one solution.

8. In prior problems, we mentioned that the graph of $y = x^2 + 1$ has no x-intercepts. Does the graph of $y = x^2 + 1$ intersect the graph of $y = x^3 + 1$?