Lesson 31: Systems of Equations

Classwork

Exploratory Challenge 1

* 1. Sketch the lines given by and on the same set of axes, and then solve the pair of equations algebraically to verify your graphical solution.
  2. Suppose the second line is replaced by the line with equation . Plot the two lines on the same set of axes, and solve the pair of equations algebraically to verify your graphical solution.
  3. Suppose the second line is replaced by the line with equation . Plot the lines on the same set of axes, and solve the pair of equations algebraically to verify your graphical solution.
  4. We have seen that a pair of lines can intersect in , , or an infinite number of points. Are there any other possibilities?

Exploratory Challenge 2

* 1. Suppose that instead of equations for a pair of lines, you were given an equation for a circle and an equation for a line. What possibilities are there for the two figures to intersect? Sketch a graph for each possibility.
  2. Graph the parabola with equation . What possibilities are there for a line to intersect the parabola? Sketch each possibility.
  3. Sketch the circle given by and the line given by on the same set of axes. One solution to the pair of equations has a value of that is easily identifiable from the sketch. What is it?
  4. Solve

Exercises 1–6

1. Draw a graph of the circle with equation .

* 1. What are the solutions to the system of circle and line when the circle is given by and the line is given by ?
  2. What happens when the line is given by ?
  3. What happens when the line is given by ?

1. By solving the equations as a system, find the points common to the line with equation and the circle with equation . Graph the line and the circle to show those points.

1. Graph the line given by and the circle given by . Find all solutions to the system of equations.
2. Graph the line given by and the circle given by . Find all solutions to the system of equations. Verify your result both algebraically and graphically.
3. Graph the line given by and the circle given by . Find all solutions to the system of equations. Verify your result both algebraically and graphically.

1. Graph the line given by and the quadratic curve given by . Find all solutions to the system of equations. Verify your result both algebraically and graphically.

Problem Set

Lesson Summary

Here are some steps to consider when solving systems of equations that represent a line and a quadratic curve.

1. Solve the linear equation for in terms of . This is equivalent to rewriting the equation in slope-intercept form. Note that working with the quadratic equation first would likely be more difficult and might cause the loss of a solution.
2. Replace in the quadratic equation with the expression involving from the slope-intercept form of the linear equation. That will yield an equation in one variable.
3. Solve the quadratic equation for .
4. Substitute into the linear equation to find the corresponding value of .
5. Sketch a graph of the system to check your solution.
6. Where do the lines given by and intersect?
7. Find all solutions to the following system of equations.  
   Illustrate with a graph.
8. Find all solutions to the following system of equations.  
   Illustrate with a graph.
9. Find all solutions to the following system of equations.  
   Illustrate with a graph.
10. Find all solutions to the following system of equations.  
    Illustrate with a graph.
11. Find all solutions to the following system of equations.  
    Illustrate with a graph.
12. If the following system of equations has two solutions, what is the value of ?  
    Illustrate with a graph.
13. If the following system of equations has exactly one solution, what is the value of ?  
    Illustrate with a graph.
14. If the following system of equations no solutions, what is the value of ?  
    Illustrate with a graph.