Lesson 28: A Focus on Square Roots

Classwork

Exercises 1–4

For Exercises 1–4, describe each step taken to solve the equation. Then, check the solution to see if it is valid. If it is not a valid solution, explain why.

1. $\sqrt{x} - 6 = 4$

   $\boldsymbol{\sqrt{x} = 10}$

   $\boldsymbol{x = 100}$

2. $\sqrt[3]{x} - 6 = 4$

   $\boldsymbol{\sqrt[3]{x} = 10}$

   $\boldsymbol{x = 1000}$

3. $\sqrt{x} + 6 = 4$

4. $\sqrt[3]{x} + 6 = 4$

Example 1

Solve the radical equation. Be sure to check your solutions.

$$\sqrt{3x + 5} - 2 = -1$$

Exercises 5–15

Solve each radical equation. Be sure to check your solutions.

5. $\sqrt{2x-3}=11$

6. $\sqrt[3]{6-x}=-3$

7. $\sqrt{x+5}-9=-12$

8. $\sqrt{4x-7}=\sqrt{3x+9}$

9. $-12\sqrt{x-6}=18$

10. $3\sqrt[3]{x+2}=12$

11. $\sqrt{x^2-5}=2$

12. $\sqrt{x^2+8x}=3$

Multiply each expression.

13. $(\sqrt{x}+2)(\sqrt{x}-2)$

14. $(\sqrt{x}+4)(\sqrt{x}+4)$

15. $(\sqrt{x-5})(\sqrt{x-5})$

Example 2

Rationalize the denominator in each expression. That is, rewrite each expression so that the fraction has a rational expression in the denominator.

a. $\frac{x-9}{\sqrt{x-9}}$

b. $\frac{x-9}{\sqrt{x}+3}$

Exercises 16–18

16. Rewrite $\frac{1}{\sqrt{x}-5}$ in an equivalent form with a rational expression in the denominator.

17. Solve the radical equation $\frac{3}{\sqrt{x+3}} = 1$. Be sure to check for extraneous solutions.

18. Without solving the radical equation $\sqrt{x+5} + 9 = 0$, how could you tell that it has no real solution?

Problem Set

1. a. If $\sqrt{x} = 9$, then what is the value of x?
   b. If $x^2 = 9$, then what is the value of x?
   c. Is there a value of x such that $\sqrt{x+5} = 0$? If yes, what is the value? If no, explain why not.
   d. Is there a value of x such that $\sqrt{x} + 5 = 0$? If yes, what is the value? If no, explain why not.

2. a. Is the statement $\sqrt{x^2} = x$ true for all x-values? Explain.
   b. Is the statement $\sqrt[3]{x^3} = x$ true for all x-values? Explain.

Rationalize the denominator in each expression.

3. $\frac{4-x}{2+\sqrt{x}}$
4. $\frac{2}{\sqrt{x-12}}$
5. $\frac{1}{\sqrt{x+3}-\sqrt{x}}$

Solve each equation and check the solutions.

6. $\sqrt{x+6} = 3$
7. $2\sqrt{x+3} = 6$
8. $\sqrt{x+3} + 6 = 3$
9. $\sqrt{x+3} - 6 = 3$
10. $16 = 8 + \sqrt{x}$
11. $\sqrt{3x-5} = 7$
12. $\sqrt{2x-3} = \sqrt{10-x}$
13. $3\sqrt{x+2} + \sqrt{x-4} = 0$
14. $\frac{\sqrt{x+9}}{4} = 3$
15. $\frac{12}{\sqrt{x+9}} = 3$
16. $\sqrt{x^2+9} = 5$
17. $\sqrt{x^2-6x} = 4$
18. $\frac{5}{\sqrt{x-2}} = 5$
19. $\frac{5}{\sqrt{x}-2} = 5$
20. $\sqrt[3]{5x-3} + 8 = 6$
21. $\sqrt[3]{9-x} = 6$

22. Consider the inequality $\sqrt{x^2+4x} > 0$. Determine whether each x-value is a solution to the inequality.
    a. $x = -10$ b. $x = -4$ c. $x = 10$ d. $x = 4$

23. Show that $\frac{a-b}{\sqrt{a}-\sqrt{b}} = \sqrt{a} + \sqrt{b}$ for all values of a and b such that $a > 0$ and $b > 0$ and $a \neq b$.

24. Without actually solving the equation, explain why the equation $\sqrt{x+1} + 2 = 0$ has no solution.