Lesson 27: Word Problems Leading to Rational Equations

Classwork

Exercise 1

1. Anne and Maria play tennis almost about every weekend. So far, Anne has won out of matches.
   1. How many matches will Anne have to win in a row to improve her winning percentage to ?
   2. How many matches will Anne have to win in a row to improve her winning percentage to ?
   3. Can Anne reach a winning percentage of ?
   4. After Anne has reached a winning percentage of by winning consecutive matches as in part (b), how many matches can she now lose in a row to have a winning percentage of ?

**Example 1**

Working together, it takes Sam, Jenna, and Francisco two hours to paint one room. When Sam is working alone, he can paint one room in hours. When Jenna works alone, she can paint one room in hours. Determine how long it would take Francisco to paint one room on his own.

Exercises 2–4

1. Melissa walks miles to the house of a friend and returns home on a bike. She averages miles per hour faster when cycling than when walking, and the total time for both trips is two hours. Find her walking speed.

1. You have liters of a juice blend that is juice.
   1. How many liters of pure juice need to be added in order to make a blend that is juice?

* 1. How many liters of pure juice need to be added in order to make a blend that is juice?

* 1. Write a rational equation that relates the desired percentage to the amount of pure juice that needs to be added to make a blend that is juice, where . What is a reasonable restriction on the set of possible values of ? Explain your answer.
  2. Suppose that you have added liters of juice to the original liters. What is the percentage of juice in this blend?
  3. Solve your equation in part (c) for the amount . Are there any excluded values of the variable ? Does this make sense in the context of the problem?

1. You have a solution containing acid and a solution containing acid.
   1. How much of the solution must you add to liter of the solution to create a mixture that is acid?

* 1. Write a rational equation that relates the desired percentage to the amount of acid solution that needs to be added to liter of acid solution to make a blend that is acid, where . What is a reasonable restriction on the set of possible values of ? Explain your answer.
  2. Solve your equation in part (b) for . Are there any excluded values of ? Does this make sense in the context of the problem?
  3. If you have added some acid solution to liter of acid solution to make a acid solution, how much of the stronger acid did you add?

Problem Set

Lesson Summary

In this lesson, we developed the students’ problem solving skills by asking them to carefully read a problem, rephrase it in a form comfortable for their own understanding, and convert fact sentences about unknown quantities into algebraic equations. Specifically, they used rational equations to model and solve some application problems and further developed their skills in working with rational expressions.

1. If inlet pipes can fill a pool in one hour and minutes, and one pipe can fill the pool in two hours and minutes on its own, how long would the other pipe take to fill the pool on its own?
2. If one inlet pipe can fill the pool in hours with the outlet drain closed, and the same inlet pipe can fill the pool in hours with the drain open, how long does it take the drain to empty the pool if there is no water entering the pool?
3. It takes minutes less time to travel miles by car at night than by day because the lack of traffic allows the average speed at night to be miles per hour faster than in the daytime. Find the average speed in the daytime.
4. The difference in the average speed of two trains is miles per hour. The slower train takes hours longer to travel miles than the faster train takes to travel miles. Find the speed of the slower train.
5. A school library spends a month on magazines. The average price for magazines bought in January was cents more than the average price in December. Because of the price increase, the school library was forced to subscribe to fewer magazines. How many magazines did the school library subscribe to in December?
6. An investor bought a number of shares of stock for . After the price dropped by per share, the investor sold all but of her shares for . How many shares did she originally buy?
7. Newton’s law of universal gravitation, , measures the force of gravity between two masses and , where is the distance between the centers of the masses, and is universal gravitational constant. Solve this equation for .
8. Suppose that .
   1. Show that when and , the value of does not depend on the value of .
   2. For which values of do these relationships have no meaning?
9. Consider the rational equation .
   1. Find the value of when and .
   2. Solve this equation for and simplify.
10. Consider an ecosystem of rabbits in a park that starts with rabbits and can sustain up to rabbits. An equation that roughly models this scenario is  
    where represents the rabbit population in year of the study.
    1. What is the rabbit population in year ? Round your answer to the nearest whole rabbit.
    2. Solve this equation for . Describe what this equation represents in the context of this problem.
    3. At what time does the population reach rabbits?
11. Suppose that Huck Finn can paint a fence in hours. If Tom Sawyer helps him pain the fence, they can do it in hours. How long would it take for Tom to paint the fence by himself?
12. Huck Finn can paint a fence in hours. After some practice, Tom Sawyer can now paint the fence in hours.
    1. How long would it take Huck and Tom to paint the fence together?
    2. Tom demands a half hour break while Huck continues to pain, and they finish the job together. How long does it take them to paint the fence?
    3. Suppose that they have to finish the fence in hours. What’s the longest break that Tom can take?