Lesson 24: Multiplying and Dividing Rational Expressions

Student Outcomes

- Students multiply and divide rational expressions and simplify using equivalent expressions.

Lesson Notes

This lesson quickly reviews the process of multiplying and dividing rational numbers using techniques students already know and translates that process to multiplying and dividing rational expressions (MP.7). This enables students to develop techniques to solve rational equations in Lesson 26 (**A-APR.D.6**). This lesson also begins developing facility with simplifying complex rational expressions, which is important for later work in trigonometry. Teachers may consider treating the multiplication and division portions of this lesson as two separate lessons.

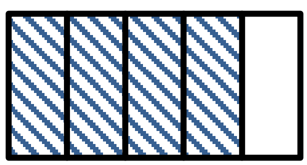
Classwork

Opening Exercise (5 minutes)

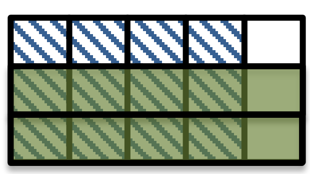
Distribute notecard-sized slips of paper to students, and ask them to shade the paper to represent the result of $\frac{2}{3} \cdot \frac{4}{5}$. Circulate around the classroom to assess student proficiency. If many students are still struggling to remember the area model after the scaffolding, present the problem to them as shown. Otherwise, allow them time to do the multiplication on their own or with their neighbor and then progress to the question of the general rule.

> *Scaffolding:*
>
> If students do not remember the area model for multiplication of fractions, have them discuss it with their neighbor. If necessary, use an example like $\frac{1}{2} \cdot \frac{1}{2}$ to see if they can scale this to the problem presented.
>
> If students are comfortable with multiplying rational numbers, omit the area model and ask them to determine the following products.
>
> - $\frac{2}{3} \cdot \frac{3}{8} = \frac{1}{4}$
> - $\frac{1}{4} \cdot \frac{5}{6} = \frac{5}{24}$
> - $\frac{4}{7} \cdot \frac{8}{9} = \frac{32}{63}$

- First, we represent $\frac{4}{5}$ by dividing our region into five vertical strips of equal area and shading 4 of the 5 parts.



- Now we need to find $\frac{2}{3}$ of the shaded area. So we divide the area horizontally into three parts of equal area, and then shade two of those parts.



- Thus, $\frac{2}{3} \cdot \frac{4}{5}$ is represented by the region that is shaded twice. Since 8 out of 15 subrectangles are shaded twice, we have $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$. With this in mind, can we create a general rule about multiplying rational numbers?

Allow students to come up with this "rule" based on the example and prior experience. Have them discuss their thoughts with their neighbor and write the rule.

If a, b, c, and d are integers with $c \neq 0$ and $d \neq 0$, then
$$\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}.$$

The rule summarized above is also valid for real numbers.

Discussion (2 minutes)

- To multiply rational expressions, we follow the same procedure we use when multiplying rational numbers: we multiply together the numerators and multiply together the denominators. We finish by reducing the product to lowest terms.

If a, b, c, and d are rational expressions with $b \neq 0$, $d \neq 0$, then
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

Lead students through Examples 1 and 2, and ask for their input at each step.

Example 1 (4 minutes)

Give students time to work on this problem and discuss their answers with a neighbor before proceeding to a whole class discussion.

Example 1

Make a conjecture about the product $\frac{x^3}{4y} \cdot \frac{y^2}{x}$. What will it be? Explain your conjecture and give evidence that it is correct.

- We begin by multiplying the numerators and denominators.
$$\frac{x^3}{4y} \cdot \frac{y^2}{x} = \frac{x^3y^2}{4yx}$$
- Identify the greatest common factor (GCF) of the numerator and denominator. The GCF of x^3y^2 and $4xy$ is xy.
$$\frac{x^3}{4y} \cdot \frac{y^2}{x} = \frac{(xy)x^2y}{4(xy)}$$

Scaffolding:

- To assist students in making the connection between rational numbers and rational expressions, show a side-by-side comparison of a numerical example from a previous lesson like the one shown.

$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$	$\frac{x^3}{4y} \cdot \frac{y^2}{x} = ?$

- If students are struggling with this example, include some others, such as
  - $\frac{x^2}{3} \cdot \frac{6}{x}$
  - $\frac{y}{x^2} \cdot \frac{y^4}{x}$

- Finally, we divide the common factor xy from the numerator and denominator to find the reduced form of the product:

$$\frac{x^3}{4y} \cdot \frac{y^2}{x} = \frac{x^2y}{4}.$$

Note that we are intentionally avoiding using the phrases "cancel xy" or "cancel the common factor" in this lesson. We want to highlight that it is division that allows us to simplify these expressions. Ambiguous words like "cancel" can lead students to simplify $\frac{\sin x}{x}$ to sin—they "canceled" the x!

It is important to understand why we are allowed to divide the numerator *and* denominator by x. The rule $\frac{na}{nb} = \frac{a}{b}$ works for rational expressions as well. Performing a simplification such as $\frac{x}{x^3y} = \frac{1}{x^2y}$ requires doing the following steps: $\frac{x}{x^3y} = \frac{x \cdot 1}{x \cdot x^2y} = \frac{x}{x} \cdot \frac{1}{x^2y} = 1 \cdot \frac{1}{x^2y} = \frac{1}{x^2y}$.

Example 2 (3 minutes)

Before walking students through the steps of this example, ask them to try to find the product using the ideas of the previous example.

Example 2

Find the following product: $\left(\frac{3x-6}{2x+6}\right) \cdot \left(\frac{5x+15}{4x+8}\right)$.

First, we can factor the numerator and denominator of each rational expression.

MP.7

- Identify any common factors in the numerator and denominator.

$$\begin{aligned}\left(\frac{3x-6}{2x+6}\right) \cdot \left(\frac{5x+15}{4x+8}\right) &= \left(\frac{3(x-2)}{2(x+3)}\right) \cdot \left(\frac{5(x+3)}{4(x+2)}\right) \\ &= \frac{15(x-2)(x+3)}{8(x+3)(x+2)}\end{aligned}$$

The GCF of the numerator and denominator is $x+3$.

Then, we can divide the common factor $(x+3)$ from the numerator and denominator and obtain the reduced form of the product.

$$\left(\frac{3x-6}{2x+6}\right) \cdot \left(\frac{5x+15}{4x+8}\right) = \frac{15(x-2)}{8(x+2)}.$$

Exercises 1–3 (5 minutes)

Students can work in pairs on the following three exercises. Circulate around the class to informally assess their understanding. For Exercise 1, listen for key points such as "factoring the numerator and denominator can help" and "multiplying rational expressions is similar to multiplying rational numbers."

Exercises 1–3

1. **Summarize what you have learned so far with your neighbor.**

   Answers will vary.

2. **Find the following product and reduce to lowest terms: $\left(\frac{2x+6}{x^2+x-6}\right)\cdot\left(\frac{x^2-4}{2x}\right)$.**

$$\left(\frac{2x+6}{x^2+x-6}\right)\cdot\left(\frac{x^2-4}{2x}\right)=\left(\frac{2(x+3)}{(x+3)(x-2)}\right)\cdot\left(\frac{(x-2)(x+2)}{2x}\right)$$
$$=\frac{2(x+3)(x-2)(x+2)}{2x(x+3)(x-2)}$$

   The factors 2, $x+3$, and $x-2$ can be divided from the numerator and the denominator in order to reduce the rational expression to lowest terms.

$$\left(\frac{2x+6}{x^2+x-6}\right)\cdot\left(\frac{x^2-4}{2x}\right)=\frac{x+2}{x}$$

3. **Find the following product and reduce to lowest terms: $\left(\frac{4n-12}{3m+6}\right)^{-2}\cdot\left(\frac{n^2-2n-3}{m^2+4m+4}\right)$.**

$$\left(\frac{4n-12}{3m+6}\right)^{-2}\cdot\left(\frac{n^2-2n-3}{m^2+4m+4}\right)=\left(\frac{3m+6}{4n-12}\right)^{2}\cdot\left(\frac{n^2-2n-3}{m^2+4m+4}\right)$$
$$=\frac{3^2(m+2)^2(n-3)(n+1)}{4^2(n-3)^2(m+2)^2}$$
$$=\frac{9(n+1)}{16(n-3)}$$

Scaffolding:

Students may need to be reminded how to interpret a negative exponent. If so, ask them to calculate these values.

- $3^{-2}=\frac{1}{3^2}=\frac{1}{9}$
- $\left(\frac{2}{5}\right)^{-3}=\left(\frac{5}{2}\right)^{3}=\frac{5^3}{2^3}=\frac{125}{8}$
- $\left(\frac{x}{y^2}\right)^{-5}=\left(\frac{y^2}{x}\right)^{5}=\frac{(y^2)^5}{x^5}=\frac{y^{10}}{x^5}$

Discussion (5 minutes)

Recall that division of numbers is equivalent to multiplication of the numerator by the reciprocal of the denominator. That is, for any two numbers a and b, where $b \neq 0$, we have

$$\frac{a}{b}=a\cdot\frac{1}{b},$$

where the number $\frac{1}{b}$ is the multiplicative inverse of b. But, what if b is itself a fraction? How do we evaluate a quotient such as $\frac{3}{5}\div\frac{4}{7}$?

- How do we evaluate $\frac{3}{5}\div\frac{4}{7}$?

Have students work in pairs to answer this and then discuss.

By our rule above, $\frac{3}{5}\div\frac{4}{7}=\frac{3}{5}\cdot\frac{1}{4/7}$. But, what is the value of $\frac{1}{4/7}$? Let x represent $\frac{1}{4/7}$, which is the multiplicative inverse of $\frac{4}{7}$. Then we have

$$x\cdot\frac{4}{7}=1$$
$$4x=7$$
$$x=\frac{7}{4}.$$

Scaffolding:

Students may be better able to generalize the procedure for dividing rational numbers by repeatedly dividing several examples, such as $\frac{1}{2}\div\frac{3}{4}$, $\frac{2}{3}\div\frac{7}{10}$, and $\frac{1}{5}\div\frac{2}{9}$. After dividing several of these examples, ask students to generalize the process (MP.8).

Since we have shown that $\frac{1}{4/7} = \frac{7}{4}$, we can continue our calculation of $\frac{3}{5} \div \frac{4}{7}$ as follows:

$$\begin{aligned}\frac{3}{5} \div \frac{4}{7} &= \frac{3}{5} \cdot \frac{1}{4/7}\\ &= \frac{3}{5} \cdot \frac{7}{4}\\ &= \frac{21}{20}.\end{aligned}$$

This same process applies to dividing rational expressions, although we might need to perform the additional step of reducing the resulting rational expression to lowest terms. Ask students to generate the rule for division of rational numbers.

> If a, b, c, and d are integers with $b \neq 0$, $c \neq 0$, and $d \neq 0$, then
> $$\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \cdot \frac{d}{b}.$$

The result summarized in the box above is also valid for real numbers.

Now that we know how to divide rational numbers, how do we extend this to divide rational expressions?

- Dividing rational expressions follows the same procedure as dividing rational numbers: we multiply the first term by the reciprocal of the second. We finish by reducing the product to lowest terms.

> **If a, b, c, and d are rational expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$, then**
> $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

Example 3 (3 minutes)

As in Example 2, ask students to apply their knowledge of rational number division to rational expressions by working on their own or with a partner. Circulate to assist and assess understanding. Once students have made attempts to divide, use the scaffolded questions to develop the concept as necessary.

> **Example 3**
>
> **Find the quotient and reduce to lowest terms:** $\frac{x^2-4}{3x} \div \frac{x-2}{2x}$.

Scaffolding:

A side-by-side comparison may help as before.

$\frac{3}{5} \div \frac{4}{7} = \frac{21}{20}$	$\frac{x^3}{4y} \div \frac{y^2}{x} = ?$

For struggling students, give

- $\frac{x^2y}{4} \div \frac{xy^2}{8} = \frac{2x}{y}$
- $\frac{3y^2}{z-1} \div \frac{12y^5}{(z-1)^2} = \frac{(z-1)}{4y^3}$.

For advanced students, give

- $\frac{x-3}{x^2+x-2} \div \frac{x^2-x-6}{x-1} = \frac{1}{(x+2)^2}$
- $\frac{x^2-2x-24}{x^2-4} \div \frac{x^2+3x-4}{x^2+x-2} = \frac{x-6}{x-2}$.

- First, we change the division of $\frac{x^2-4}{3x}$ by $\frac{x-2}{2x}$ into multiplication of $\frac{x^2-4}{3x}$ by the multiplicative inverse of $\frac{x-2}{2x}$.

$$\frac{x^2-4}{3x} \div \frac{x-2}{2x} = \frac{x^2-4}{3x} \cdot \frac{2x}{x-2}$$

- Then, we perform multiplication as in the previous examples and exercises. That is, we factor the numerator and denominator and divide any common factors present in both the numerator and denominator.

$$\begin{aligned}\frac{x^2-4}{3x} \div \frac{x-2}{2x} &= \frac{(x-2)(x+2)}{3x} \cdot \frac{2x}{x-2} \\ &= \frac{2(x+2)}{3}\end{aligned}$$

Exercise 4 (3 minutes)

Allow students to work in pairs or small groups to evaluate the following quotient.

Exercises 4–5

4. **Find the quotient and reduce to lowest terms: $\frac{x^2-5x+6}{x+4} \div \frac{x^2-9}{x^2+5x+4}$.**

$$\begin{aligned}\frac{x^2-5x+6}{x+4} \div \frac{x^2-9}{x^2+5x+4} &= \frac{x^2-5x+6}{x+4} \cdot \frac{x^2+5x+4}{x^2-9} \\ &= \frac{(x-3)(x-2)}{x+4} \cdot \frac{(x+4)(x+1)}{(x-3)(x+3)} \\ &= \frac{(x-2)(x+1)}{(x+3)}\end{aligned}$$

Discussion (4 minutes)

What do we do when the numerator and denominator of a fraction are themselves fractions? We call a fraction that contains fractions a *complex fraction*. Remind students that the fraction bar represents division, so a complex fraction represents division between rational expressions.

Allow students the opportunity to simplify the following complex fraction.

$$\frac{12/49}{27/28}$$

Allow students to struggle with the problem before discussing solution methods.

$$\begin{aligned}\frac{12/49}{27/28} &= \frac{12}{49} \div \frac{27}{28} \\ &= \frac{12}{49} \cdot \frac{28}{27} \\ &= \frac{3 \cdot 4}{7 \cdot 7} \cdot \frac{4 \cdot 7}{3^3} \\ &= \frac{4^2}{7 \cdot 3^2} \\ &= \frac{16}{63}\end{aligned}$$

Notice that in simplifying the complex fraction above, we are merely performing division of rational numbers, and we already know how to do that. Since we already know how to divide rational expressions, we can also simplify rational expressions whose numerators and denominators are rational expressions.

Exercise 5 (4 minutes)

Allow students to work in pairs or small groups to simplify the following rational expression.

> *Scaffolding:*
>
> For struggling students, give a simpler example, such as
>
> $$\frac{\left(\frac{2x}{3y}\right)}{\left(\frac{6x}{4y^2}\right)} = \frac{2x}{3y} \div \frac{6x}{4y^2}$$
> $$= \frac{2x}{3y} \cdot \frac{4y^2}{6x}$$
> $$= \frac{4}{9}y.$$

5. Simplify the rational expression.

$$\frac{\left(\frac{x+2}{x^2-2x-3}\right)}{\left(\frac{x^2-x-6}{x^2+6x+5}\right)}$$

$$\frac{\frac{x+2}{x^2-2x-3}}{\frac{x^2-x-6}{x^2+6x+5}} = \frac{x+2}{x^2-2x-3} \div \frac{x^2-x-6}{x^2+6x+5}$$

$$= \frac{x+2}{x^2-2x-3} \cdot \frac{x^2+6x+5}{x^2-x-6}$$

$$= \frac{x+2}{(x-3)(x+1)} \cdot \frac{(x+5)(x+1)}{(x-3)(x+2)}$$

$$= \frac{x+5}{(x-3)^2}$$

Closing (3 minutes)

Ask students to summarize the important parts of the lesson in writing, to a partner, or as a class. Use this as an opportunity to informally assess understanding of the lesson. In particular, ask students to articulate the processes for multiplying and dividing rational expressions and simplifying complex rational expressions either verbally or symbolically.

> Lesson Summary
>
> **In this lesson we extended multiplication and division of rational numbers to multiplication and division of rational expressions.**
>
> - **To multiply two rational expressions, multiply the numerators together and multiply the denominators together, and then reduce to lowest terms.**
> - **To divide one rational expression by another, multiply the first by the multiplicative inverse of the second, and reduce to lowest terms.**
> - **To simplify a complex fraction, apply the process for dividing one rational expression by another.**

Exit Ticket (4 minutes)

Name ____ Date ____________________

Lesson 24: Multiplying and Dividing Rational Expressions

Exit Ticket

Perform the indicated operations and reduce to lowest terms.

1. $\frac{x-2}{x^2+x-2} \cdot \frac{x^2-3x+2}{x+2}$

2. $\frac{\left(\frac{x-2}{x^2+x-2}\right)}{\left(\frac{x^2-3x+2}{x+2}\right)}$

Exit Ticket Sample Solutions

Perform the indicated operations and reduce to lowest terms.

1. $\frac{x-2}{x^2+x-2} \cdot \frac{x^2-3x+2}{x+2}$

$$\frac{x-2}{x^2+x-2} \cdot \frac{x^2-3x+2}{x+2} = \frac{x-2}{(x-1)(x+2)} \cdot \frac{(x-1)(x-2)}{x+2}$$
$$= \frac{(x-2)^2}{(x+2)^2}$$

2. $\frac{\left(\frac{x-2}{x^2+x-2}\right)}{\left(\frac{x^2-3x+2}{x+2}\right)}$

$$\frac{\left(\frac{x-2}{x^2+x-2}\right)}{\left(\frac{x^2-3x+2}{x+2}\right)} = \frac{x-2}{x^2+x-2} \div \frac{x^2-3x+2}{x+2}$$
$$= \frac{x-2}{x^2+x-2} \cdot \frac{x+2}{x^2-3x+2}$$
$$= \frac{x-2}{(x-1)(x+2)} \cdot \frac{x+2}{(x-2)(x-1)}$$
$$= \frac{1}{(x-1)^2}$$

Problem Set Sample Solutions

1. **Complete the following operations:**

   a. Multiply $\frac{1}{3}(x-2)$ by 9.

   $3x-6$

   b. Divide $\frac{1}{4}(x-8)$ by $\frac{1}{12}$.

   $3x-24$

   c. Multiply $\frac{1}{4}\left(\frac{1}{3}x+2\right)$ by 12.

   $x+6$

   d. Divide $\frac{1}{3}\left(\frac{2}{5}x-\frac{1}{5}\right)$ by $\frac{1}{15}$.

   $2x-1$

   e. Multiply $\frac{2}{3}\left(2x+\frac{2}{3}\right)$ by $\frac{9}{4}$.

   $3x+1$

   f. Multiply $0.03(4-x)$ by 100.

   $12-3x$

2. **Simplify each of the following expressions.**

   a. $\left(\frac{a^3b^2}{c^2d^2} \cdot \frac{c}{ab}\right) \div \frac{a}{c^2d^3}$

   $abcd$

   b. $\frac{a^2+6a+9}{a^2-9} \cdot \frac{3a-9}{a+3}$

   3

   c. $\frac{6x}{4x-16} \div \frac{4x}{x^2-16}$

   $\frac{3(x+4)}{8}$

   d. $\frac{3x^2-6x}{3x+1} \cdot \frac{x+3x^2}{x^2-4x+4}$

   $\frac{3x^2}{x-2}$

   e. $\frac{2x^2-10x+12}{x^2-4} \cdot \frac{2+x}{3-x}$

   -2

   f. $\frac{a-2b}{a+2b} \div (4b^2-a^2)$

   $-\frac{1}{(a+2b)^2}$

g. $\frac{d+c}{c^2+d^2} \div \frac{c^2-d^2}{d^2-dc}$

$-\frac{d}{c^2+d^2}$

h. $\frac{12a^2-7ab+b^2}{9a^2-b^2} \div \frac{16a^2-b^2}{3ab+b^2}$

$\frac{b}{4a+b}$

i. $\left(\frac{x-3}{x^2-4}\right)^{-1} \cdot \left(\frac{x^2-x-6}{x-2}\right)$

$(x+2)^2$

j. $\left(\frac{x-2}{x^2+1}\right)^{-3} \div \left(\frac{x^2-4x+4}{x^2-2x-3}\right)$

$\frac{(x-3)(x+1)(x^2+1)^3}{(x-2)^5}$

k. $\frac{6x^2-11x-10}{6x^2-5x-6} \cdot \frac{6-4x}{25-20x+4x^2}$

$-\frac{2}{2x-5}$, *or* $\frac{2}{5-2x}$

l. $\frac{3x^3-3a^2x}{x^2-2ax+a^2} \cdot \frac{a-x}{a^3x+a^2x^2}$

$-\frac{3}{a^2}$

3. Simplify the following complex rational expressions.

a. $\frac{\left(\frac{4a}{6b^2}\right)}{\left(\frac{20a^3}{12b}\right)}$ $\quad \frac{2}{5a^2b}$

b. $\frac{\left(\frac{x-2}{x^2-1}\right)}{\left(\frac{x^2-4}{x-6}\right)}$ $\quad \frac{x-6}{(x+2)(x^2-1)}$

c. $\frac{\left(\frac{x^2+2x-3}{x^2+3x-4}\right)}{\left(\frac{x^2+x-6}{x+4}\right)}$ $\quad \frac{1}{x-2}$

4. Suppose that $x = \frac{t^2+3t-4}{3t^2-3}$ and $y = \frac{t^2+2t-8}{2t^2-2t-4}$, for $t \neq 1$, $t \neq -1$, $t \neq 2$, and $t \neq -4$. Show that the value of x^2y^{-2} does not depend on the value of t.

$$\begin{aligned}
x^2y^{-2} &= \left(\frac{t^2+3t-4}{3t^2-3}\right)^2\left(\frac{t^2+2t-8}{2t^2-2t-4}\right)^{-2} \\
&= \left(\frac{t^2+3t-4}{3t^2-3}\right)^2 \div \left(\frac{t^2+2t-8}{2t^2-2t-4}\right)^2 \\
&= \left(\frac{t^2+3t-4}{3t^2-3}\right)^2\left(\frac{2t^2-2t-4}{t^2+2t-8}\right)^2 \\
&= \left(\frac{(t-1)(t+4)}{3(t-1)(t+1)}\right)^2\left(\frac{2(t-2)(t+1)}{(t-2)(t+4)}\right)^2 \\
&= \frac{4(t-1)^2(t+4)^2(t-2)^2(t+1)^2}{9(t-1)^2(t+1)^2(t-2)^2(t+4)^2} \\
&= \frac{4}{9}
\end{aligned}$$

Since $x^2y^{-2} = \frac{4}{9}$, the value of x^2y^{-2} does not depend on t.

5. Determine which of the following numbers is larger without using a calculator, $\frac{15^{16}}{16^{15}}$ or $\frac{20^{24}}{24^{20}}$. (Hint: We can compare two positive quantities a and b by computing the quotient $\frac{a}{b}$. If $\frac{a}{b} > 1$, then $a > b$. Likewise, if $0 < \frac{a}{b} < 1$, then $a < b$.)

$$\begin{aligned}\frac{15^{16}}{16^{15}} \div \frac{20^{24}}{24^{20}} &= \frac{15^{16}}{16^{15}} \cdot \frac{24^{20}}{20^{24}} \\ &= \frac{3^{16} \cdot 5^{16}}{(2^4)^{15}} \cdot \frac{(2^3)^{20} \cdot 3^{20}}{(2^2)^{24} \cdot 5^{24}} \\ &= \frac{2^{60} \cdot 3^{36} \cdot 5^{16}}{2^{108} \cdot 5^{24}} \\ &= \frac{3^{36}}{2^{48} \cdot 5^8} \\ &= \frac{9^{18}}{2^{40} \cdot 10^8} \\ &= \left(\frac{9}{10}\right)^8 \cdot \left(\frac{9^{10}}{2^{40}}\right) \\ &= \left(\frac{9}{10}\right)^8 \cdot \left(\frac{9}{16}\right)^{10}\end{aligned}$$

Since $\frac{9}{16} < 1$, and $\frac{9}{10} < 1$, we know that $\left(\frac{9}{10}\right)^8 \cdot \left(\frac{9}{16}\right)^{10} < 1$. Thus, $\frac{15^{16}}{16^{15}} \div \frac{20^{24}}{24^{20}} < 1$, and we know that $\frac{15^{16}}{16^{15}} < \frac{20^{24}}{24^{20}}$.

6. (Optional) One of two numbers can be represented by the rational expression $\frac{x-2}{x}$, where $x \neq 0$ and $x \neq 2$.

   a. Find a representation of the second number if the product of the two numbers is 1.

   Let the second number be y. Then $\left(\frac{x-2}{x}\right) \cdot y = 1$, so we have

$$\begin{aligned}y &= 1 \div \left(\frac{x-2}{x}\right) \\ &= 1 \cdot \left(\frac{x}{x-2}\right) \\ &= \frac{x}{x-2}.\end{aligned}$$

   b. Find a representation of the second number if the product of the two numbers is 0.

   Let the second number be z. Then $\left(\frac{x-2}{x}\right) \cdot z = 0$, so we have

$$\begin{aligned}z &= 0 \div \left(\frac{x-2}{x}\right) \\ &= 0 \cdot \left(\frac{x}{x-2}\right) \\ &= 0.\end{aligned}$$