Lesson 19: The Remainder Theorem

Classwork

Exercises 1–3

1. Consider the polynomial function $f\left(x\right)=3x^{2}+8x-4$.

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| * 1. Divide $f$ by $x-2$.
 | * 1. Find $f(2).$
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1. Consider the polynomial function $g\left(x\right)=x^{3}-3x^{2}+6x+8$.

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| * 1. Divide $g$ by $x+1$.
 | * 1. Find $g(-1)$.
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1. Consider the polynomial function $h\left(x\right)=x^{3}+2x-3$.

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| * 1. Divide $h$ by $x-3$.
 | * 1. Find $h(3)$.
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Exercises 4–6

1. Consider the polynomial $P\left(x\right)=x^{3}+kx^{2}+x+6$.
	1. Find the value of $k$ so that $x+1$ is a factor of $P$.
	2. Find the other two factors of $P$ for the value of $k$ found in part (a).
2. Consider the polynomial $P\left(x\right)=x^{4}+3x^{3}-28x^{2}-36x+144$.
	1. Is $1$ a zero of the polynomial $P$?
	2. Is $x+3$ one of the factors of $P$?



* 1. The graph of $P$ is shown to the right. What are the zeros of $P$?
	2. Write the equation of $P$ in factored form.
1. Consider the graph of a degree $5$ polynomial shown to the right, with $x$-intercepts $-4$, $-2$, $1$, $3,$ and $5$.
	1. Write a formula for a possible polynomial function that the graph represents using $c$ as constant factor.
	2. Suppose the $y$-intercept is $-4$. Adjust your function to fit the $y$-intercept by finding the constant factor $c$.

Problem Set

Lesson Summary

**Remainder Theorem:**

Let $P$ be a polynomial function in $x$, and let $a$ be any real number. Then there exists a unique polynomial function $q$ such that the equation

$$P(x)=q(x)(x-a)+P(a)$$

is true for all $x$. That is, when a polynomial is divided by $(x-a)$, the remainder is the value of the polynomial evaluated at $a$.

**Factor Theorem:**

Let $P$ be a polynomial function in $x$, and let $a$ be any real number. If $a$ is a zero of $P$, then $(x-a)$ is a factor of $P$.

Example: If $P\left(x\right)=x^{2}-3$ and $a=4$, then $P(x)=(x+4)(x-4)+13$ where $q(x)=x+4$ and $P(4)=13$.

Example: If $P(x)=x^{3}-5x^{2}+3x+9$, then $P\left(3\right)=27-45+9+9=0$, so $(x-3)$ is a factor of $P.$

1. Use the Remainder Theorem to find the remainder for each of the following divisions.
	1. $\left(x^{2}+3x+1\right)÷\left(x+2\right)$
	2. $(x^{3}-6x^{2}-7x+9)÷(x-3)$
	3. $(32x^{4}+24x^{3}-12x^{2}+2x+1)÷\left(x+1\right)$
	4. $(32x^{4}+24x^{3}-12x^{2}+2x+1)÷\left(2x-1\right)$
	5. Hint for part (d): Can you rewrite the division expression so that the divisor is in the form $\left(x-c\right)$ for some constant $c$?
2. Consider the polynomial $P\left(x\right)=x^{3}+6x^{2}-8x-1$. Find $P(4)$ in two ways.
3. Consider the polynomial function $P\left(x\right)=2x^{4}+3x^{2}+12$.
	1. Divide $P$ by $x+2$ and rewrite $P$ in the form $(divisor)(quotient)+remainder$.
	2. Find $P(-2)$.
4. Consider the polynomial function $P\left(x\right)=x^{3}+42$.
	1. Divide $P$ by $x-4$ and rewrite $P$ in the form $(divisor)(quotient)+remainder$.
	2. Find $P(4)$.
5. Explain why for a polynomial function $P$, $P(a)$ is equal to the remainder of the quotient of $P$ and $x-a$.
6. Is $x-5$ a factor of the function $f\left(x\right)=x^{3}+x^{2}-27x-15$? Show work supporting your answer.
7. Is $x+1$ a factor of the function $f\left(x\right)=2x^{5}-4x^{4}+9x^{3}-x+13$? Show work supporting your answer.
8. A polynomial function $p$has zeros of $2$, $2$,$ -3$,$ -3$,$ -3$, and $4$. Find a possible formula for $p$ and state its degree. Why is the degree of the polynomial not $3$?
9. Consider the polynomial function $P\left(x\right)=x^{3}-8x^{2}-29x+180$.
	1. Verify that $P(9)=0$. Since $P(9)=0$, what must one of the factors of $P$ be?
	2. Find the remaining two factors of $P$.
	3. State the zeros of $P$.
	4. Sketch the graph of $P.$



1. Consider the polynomial function $P\left(x\right)=2x^{3}+3x^{2}-2x-3$.
	1. Verify that $P(-1)=0$. Since $P(-1)=0$, what must one of the factors of $P$ be?
	2. Find the remaining two factors of $P$.
	3. State the zeros of $P$.
	4. Sketch the graph of $P.$



1. The graph to the right is of a third degree polynomial function $f$.
	1. State the zeros of $f$.
	2. Write a formula for $f$ in factored form using $c$ for the constant factor.
	3. Use the fact that $f(-4)=-54$ to find the constant factor.
	4. Verify your equation by using the fact that $f(1)=11$.
2. Find the value of $k$ so that $(x^{3}-kx^{2}+2)÷(x-1)$ has remainder $8$.
3. Find the value $k$ so that $(kx^{3}+x-k)÷(x+2$) has remainder $16$.
4. Show that $x^{51}-21x+20$ is divisible by $x-1$.
5. Show that $x+1$ is a factor of $19x^{42}+18x-1$*.*

Write a polynomial function that meets the stated conditions.

1. The zeros are $-2$ and $1$.
2. The zeros are $-1$, $2$, and $7.$
3. The zeros are$ -\frac{1}{2}$ and$ \frac{3}{4}$.
4. The zeros are $-\frac{2}{3}$ and $5$, and the constant term of the polynomial is $-10.$
5. The zeros are $2$ and $-\frac{3}{2}$, the polynomial has degree $3$ and there are no other zeros.