



Lesson 14: Graphing Factored Polynomials

Student Outcomes

- Students will use the factored forms of polynomials to find zeros of a function.
- Students will use the factored forms of polynomials to sketch the components of graphs between zeros.

Lesson Notes

In this lesson, we use the factored form of polynomials to identify important aspects of the graphs of polynomial functions and, therefore, important aspects of the situations they model. Using the factored form, we will identify zeros of the polynomial (and thus x -intercepts of the graph of the polynomial function) and see how to sketch a graph of the polynomial functions by examining what happens between the x -intercepts. We also introduce the concepts of relative minima and maxima and determining the possible degree of the polynomial by noting the number of relative extrema in a graph. We include definitions of relevant vocabulary at the end of the lesson for reference so that each definition can be used appropriately.

MP.5
&
MP.7

The use of a graphing utility is recommended for some examples in this lesson to encourage students to focus on understanding the structure of the polynomials without the tedium of repeated graphing by hand.

Opening Exercise (9 minutes)

Prompt the students to answer part (a) of the Opening Exercise independently or in pairs before continuing with the scaffolded questions.

Opening Exercise

An engineer is designing a roller coaster for younger children and has tried some functions to model the height of the roller coaster during the first 300 yards. She came up with the following function to describe what she believes would make a fun start to the ride:

$$H(x) = -3x^4 + 21x^3 - 48x^2 + 36x,$$

where $H(x)$ is the height of the roller coaster (in yards) when the roller coaster is $100x$ yards from the beginning of the ride. Answer the following questions to help determine at which distances from the beginning of the ride the roller coaster is at its lowest height.

- Does this function describe a roller coaster that would be fun to ride? Explain.
Yes, the roller coaster quickly goes to the top then drops you down. This looks like a fun ride.
OR
No, I don't like roller coasters that climb steeply, and this one goes nearly straight up.
- Can you see any obvious x -values from the equation where the roller coaster is at height 0?
The height is zero when x is 0 because at that value each term is equal to 0.

Scaffolding:

- Consider beginning the class by reviewing graphs of simpler functions modeling "simple roller coasters," such as $G(x) = -x^2 + 4x$.
- A more visual approach may be taken by first describing and analyzing the graph of H before connecting each concept to the algebra associated with the function. Pose questions such as "When is the roller coaster going up? Going down? How many times does the roller coaster touch the bottom?"

MP.3

MP.5

MP.3

- c. Using a graphing utility, graph the function H on the interval $0 \leq x \leq 3$, and identify when the roller coaster is 0 yards off the ground.

The lowest points of the graph on $0 \leq x \leq 3$ are when the x -value satisfies $H(x) = 0$, which occurs when x is 0, 2, and 3.

- d. What do the x -values you found in part (c) mean in terms of distance from the beginning of the ride?

The distances represent 0 yards, 200 yards, and 300 yards, respectively.

- e. Why do roller coasters always start with the largest hill first?

So they can build up speed from gravity to help propel the cars through the rest of the track.

- f. Verify your answers to part (c) by factoring the polynomial function H .

Some students may need some hints or guidance with factoring.

$$\begin{aligned} H(x) &= -3x^4 + 21x^3 - 48x^2 + 36x \\ &= -3x(x^3 - 7x^2 + 16x - 12) \end{aligned}$$

From the graph, we suspect that $(x - 3)$ is a factor; using long division, we obtain

$$\begin{aligned} H(x) &= -3x(x - 3)(x^2 - 4x + 4) \\ &= -3x(x - 3)(x - 2)(x - 2) \\ &= -3x(x - 3)(x - 2)^2. \end{aligned}$$

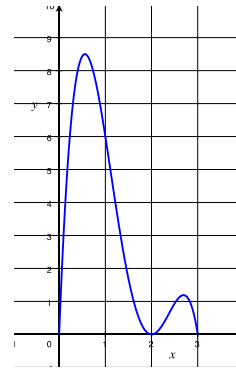
The solutions to the equation $H(x) = 0$ are 0, 2, and 3. Therefore, the roller coaster is at the bottom at 0 yards, 200 yards, and 300 yards from the start of the ride.

- g. How do you think the engineer came up with the function for this model?

Let students discuss this question in groups or as a whole class. The following conclusion should be made: To start at height 0 yards and end 300 yards later at height 0 yards, she multiplied x by $x - 3$ (to create zeros at 0 and 3). To create the bottom of the hill at 200 yards, she multiplied this function by $(x - 2)^2$. She needed to multiply by -3 to guarantee the roller coaster shape and to adjust the overall height of the roller coaster.

- h. What is wrong with this roller coaster model at distance 0 yards and 300 yards? Why might this not initially bother the engineer when she is first designing the track?

The model appears to abruptly start at 0 yards and abruptly end at 300 yards. In fact, the roller coaster looks as if it will crash into the ground at 300 yards! The engineer may be planning to "smooth" out the track later at 0 yards and 300 yards after she has selected the overall shape of the roller coaster.



Discussion (4 minutes)

By manipulating a polynomial function into its factored form, we can identify the zeros of the function as well as identify the general shape of the graph. Thinking about the Opening Exercise, what else can we say about the polynomial function and its graph?

Scaffolding:

Encourage struggling learners to graph the original and the factored forms using a graphing utility to confirm that they are the same.

- The degree of the polynomial function H is 4. How can you find the degree of the function from its factored form?
 - Add the highest degree term from each factor:
 - -3 is a degree 0 factor
 - x is degree 1 factor
 - $x - 3$ is degree 1 factor
 - $(x - 2)^2$ is a degree 2 factor, since $(x - 2)^2 = (x - 2)(x - 2)$.
 - Thus, $0 + 1 + 1 + 2 = 4$.
- How many x -intercepts does the graph of the polynomial function have?
 - For this graph, there are three: $(0,0)$, $(2,0)$, and $(3,0)$.

You may want to include a discussion that the zeros of a function correspond to the x -intercepts of the graph of the function.

- Note that there are four factors, but only three x -intercepts. Why is that?
 - Two of the factors are the same.

Remind students that the x -intercepts of a graph $y = f(x)$ are solutions to the equation $f(x) = 0$. Values of r that satisfy $f(r) = 0$ are called zeros (or roots) of the function. Some of these zeros may be repeated.

- Can you make one change to the polynomial function such that the new graph would have four x -intercepts?
 - Change one of the $(x - 2)$ factors to $(x - 1)$, for example.

Scaffolding:

- For advanced learners, consider challenging students to construct a variety of functions to meet different criteria (such as three factors and no x -intercepts or four factors with two x -intercepts).
- Students may enjoy challenging each other by trying to guess the equation that goes with the graph of their classmates.

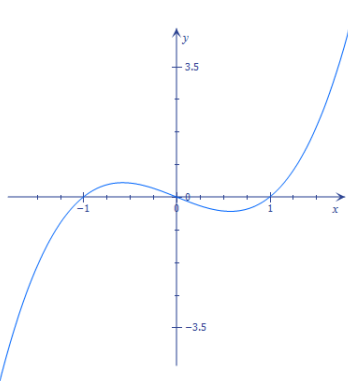
Example 1 (10 minutes)

We are now going to examine a few polynomial functions in factored form and compare the zeros of the function to the graph of the function on the calculator. Help students with part (a), and ask them to do part (b) on their own.

Example 1

Graph each of the following polynomial functions. What are the function's zeros (counting multiplicities)? What are the solutions to $f(x) = 0$? What are the x -intercepts of the graph of the function? How does the degree of the polynomial function compare to the x -intercepts of the graph of the function?

a. $f(x) = x(x - 1)(x + 1)$



Zeros: $-1, 0, 1$

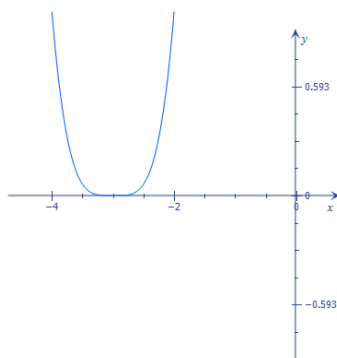
Solutions to $f(x) = 0$: $-1, 0, 1$

x -intercepts: $-1, 0, 1$

The degree is 3, which is the same as the number of x -intercepts.

Before graphing the next equation, ask students where they think the graph of f will cross the x -axis and how the repeated factor will affect the graph. After graphing, students may need to trace near $x = -3$ depending on the graphing window to obtain a clear picture of the x -intercept.

b. $f(x) = (x + 3)(x + 3)(x + 3)(x + 3)$



Zeros: $-3, -3, -3, -3$ (repeated zero)

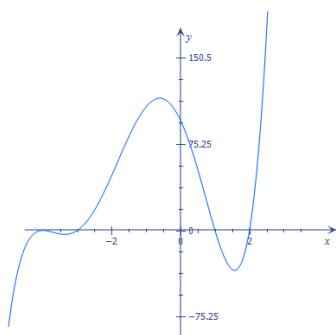
Solutions to $f(x) = 0$: -3

x -intercept: -3

The degree is 4, which is greater than the number of x -intercepts.

By now, students should have an idea of what to expect in part (c). It may be worth noting the differences in the end behavior of the graphs, which will be explored further in Lesson 15. Discuss the degree of each polynomial.

c. $f(x) = (x - 1)(x - 2)(x + 3)(x + 4)(x + 4)$



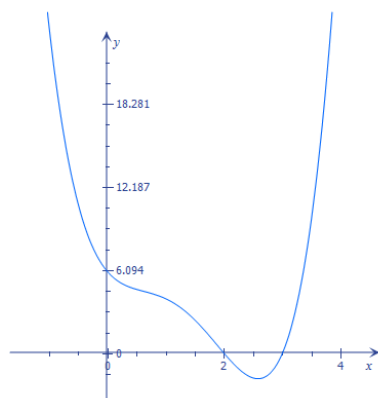
Zeros: $-4, -4, -3, 1, 2$

Solutions to $f(x) = 0$: $-4, -3, 1, 2$

x -intercepts: $-4, -3, 1, 2$

The degree is 5, which is greater than the number of x -intercepts.

d. $f(x) = (x^2 + 1)(x - 2)(x - 3)$



Zeros: $2, 3$

Solutions to $f(x) = 0$: $2, 3$

x -intercepts: $2, 3$

The degree is 4, which is greater than the number of x -intercepts.

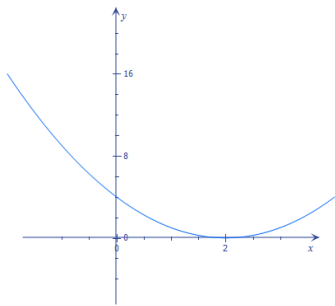
- Why is the factor $x^2 + 1$ never zero and how does this affect the graph of f ?

(At this point in the module, all polynomial functions are defined from the real numbers to the real numbers; hence, the functions can have only real number zeros. We will extend polynomial functions to the domain of complex numbers later, and then it will be possible to have complex solutions.)

- For real numbers x , the value of x^2 is always greater than or equal to zero, so $x^2 + 1$ will always be strictly greater than zero. Thus, $x^2 + 1 \neq 0$ for all real numbers x . Since there can be no x -intercept from this factor, the graph of f can have at most two x -intercepts.

If there is time, consider graphing the functions for parts (e)–(h) on the board and asking students to match the functions to the graphs. Encourage students to use a graphing utility to graph their guesses, talk about the differences between guesses and the actual graph, and what may cause them in each case.

e. $f(x) = (x - 2)^2$



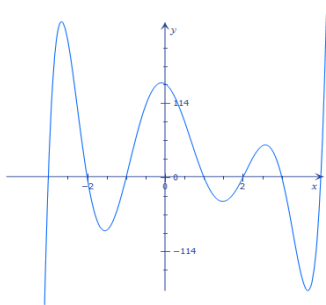
Zeros: 2, 2

Solutions to $f(x) = 0$: 2

x -intercepts: 2

The degree is 2, which is greater than the number of x -intercepts.

f. $f(x) = (x - 1)(x + 1)(x - 2)(x + 2)(x - 3)(x + 3)(x - 4)$



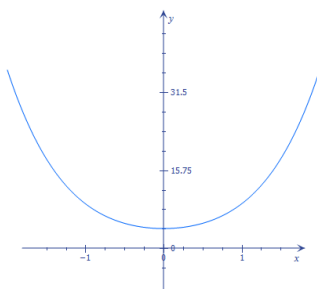
Zeros: 1, -1, 2, -2, 3, -3, 4

Solutions to $f(x) = 0$: 1, -1, 2, -2, 3, -3, 4

x -intercepts: 1, -1, 2, -2, 3, -3, 4

The degree is 7, which is equal to the number of x -intercepts.

g. $f(x) = (x^2 + 2)^2$



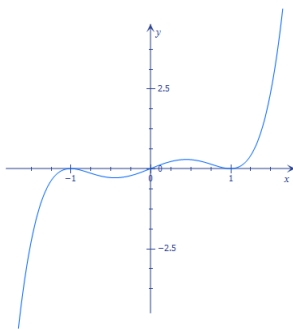
Zeros: None

Solutions to $f(x) = 0$: No solutions

x -intercepts: No x -intercepts

The degree is 4, which is greater than the number of x -intercepts.

h. $f(x) = (x + 1)^2(x - 1)^2x$



Zeros: $-1, -1, 1, 1, 0$

Solutions to $f(x) = 0$: $-1, 0, 1$

x -intercepts: $-1, 0, 1$

The degree is 5, which is greater than the number of x -intercepts.

Discussion (1 minutes)

Ask students to summarize what they have learned so far, either in writing or with a partner. Check for understanding of the concepts, and help students reach the following conclusions if they do not do so on their own.

- The x -intercepts in the graph of a function correspond to the solutions to the equation $f(x) = 0$ and correspond to the number of distinct zeros of the function (but the x -intercepts do not help us to determine the multiplicity of a given zero).
- The graph of a polynomial function of degree n has at most n x -intercepts but may have fewer.
- A polynomial function whose graph has m x -intercepts is at least a degree m polynomial.

Example 2 (8 minutes)

Lead the students through the questions in order to arrive at a sketch of the final graph. The main point of this exercise is that if we know the x -intercepts of a polynomial function, we can sketch a fairly accurate graph of the function by just checking to see if the function is positive or negative at a few points. We are not graphing by plotting points and connecting the dots but by applying what we know of polynomial functions.

Give time for students to work through parts (a) and (b) in pairs or small groups before continuing with the discussion in parts (c)-(i). When sketching the graph in part (j), it is important to let the students know that we cannot pinpoint exactly the high and low points on the graph—the relative maximum and minimum values. For this reason, omit a scale on the y -axis in the sketch.

Example 2

Consider the function $f(x) = x^3 - 13x^2 + 44x - 32$.

- a. Use the fact that $x - 4$ is a factor of f to factor this polynomial.

Using polynomial division and then factoring, $f(x) = (x - 4)(x^2 - 9x + 8) = (x - 4)(x - 8)(x - 1)$.

- b. Find the x -intercepts for the graph of f .

The x -intercepts are 1, 4, and 8.

- c. At which x -values can the function change from being positive to negative or from negative to positive?

Only at the x -intercepts 1, 4, and 8.

- d. To sketch a graph of f , we need to consider whether the function is positive or negative on the four intervals $x < 1$, $1 < x < 4$, $4 < x < 8$, and $x > 8$. Why is that?

The function can only change sign at the x -intercepts; therefore, on each of those intervals, the graph will always be above or always be below the axis.

- e. How can we tell if the function is positive or negative on an interval between x -intercepts?

Evaluate the function at a single point in that interval. Since the function is either always positive or always negative between x -intercepts, checking a single point will indicate behavior on the entire interval.

- f. For $x < 1$, is the graph above or below the x -axis? How can you tell?

Since $f(0) = -32$ is negative, the graph is below the x -axis for $x < 1$.

- g. For $1 < x < 4$, is the graph above or below the x -axis? How can you tell?

Since $f(2) = 12$ is positive, the graph is above the x -axis for $1 < x < 4$.

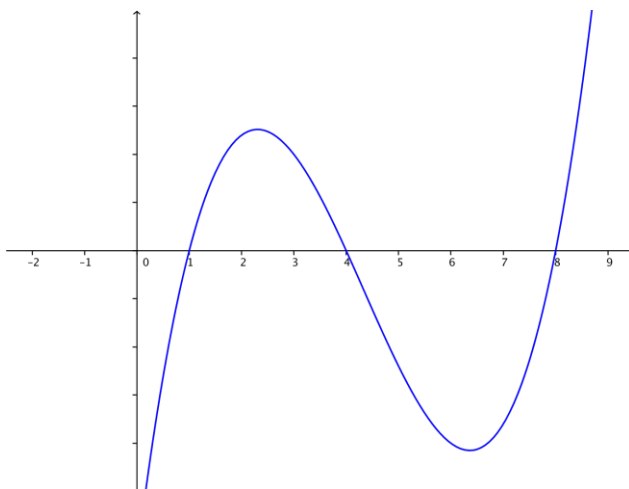
- h. For $4 < x < 8$, is the graph above or below the x -axis? How can you tell?

Since $f(5) = -12$ is negative, the graph is below the x -axis for $4 < x < 8$.

- i. For $x > 8$, is the graph above or below the x -axis? How can you tell?

Since $f(10) = 108$ is positive, the graph is above the x -axis for $x > 8$.

- j. Use the information generated in parts (f)–(i) to sketch a graph of f .



- k. Graph $y = f(x)$ on the interval from $[0, 9]$ using a graphing utility, and compare your sketch with the graph generated by the graphing utility.

Discussion (6 minutes)

- Let's examine the interval $[1,4]$ in the graph of $f(x) = x^3 - 13x^2 + 44x - 32$ above. Is there a number c in that interval where the value $f(c)$ is greater than or equal to any other value of the function on that interval? Do we know exactly where that is?
 - There is a value of c such that $f(c)$ is greater than or equal to the other values. It seems that $2 < c < 2.5$, but we do not know its exact value.

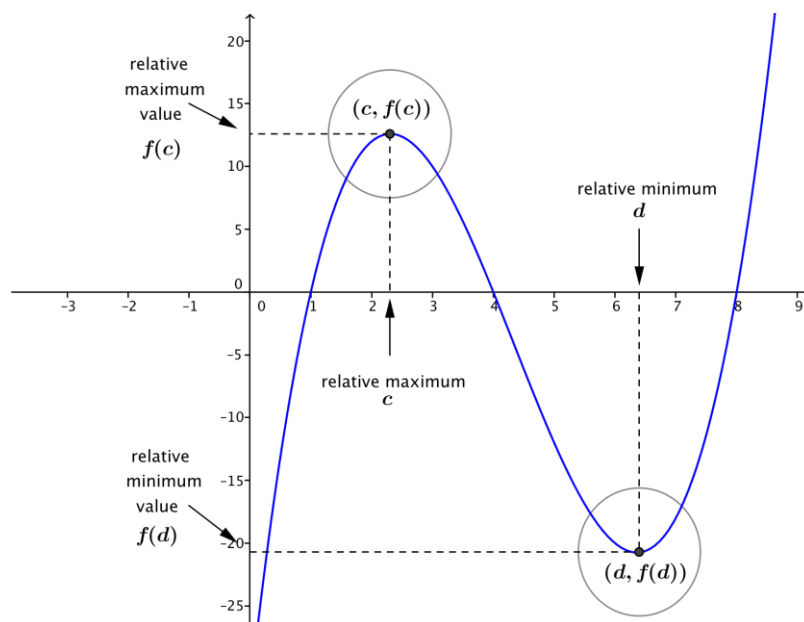
You might mention that we can find c exactly using calculus, but this is a topic for another class. For now, you can point out that we can always find relative maxima or relative minima of quadratic functions—they occur at the x -value of the vertex.

- We call the number c a **relative maximum**. The *relative maximum value*, $f(c)$, may not be the greatest overall value of the function, but there is a circle around $(c, f(c))$ so that there is no point on the graph higher than $(c, f(c))$ in that circle.
- Similarly, a **relative minimum** is a number d in the domain of f such that there is a circle around the point $(d, f(d))$ so that there is no point on the graph lower than $(d, f(d))$ in that circle. The *relative minimum value* is $f(d)$.
- Show the relative maxima and relative minima on the graph. The terminology can be confusing, but the image below clarifies the distinction between the relative maximum and the relative minimum value. Point out that there are values of the function that are larger than $f(c)$, such as $f(9)$, but that $f(c)$ is the highest value among the “neighbors” of c .

Scaffolding:

For English Language Learners, the term “relative” may need some additional instruction and practice to help differentiate it from other uses of this word.

It may help to think of the other points in the interval containing the relative maxima as all being related, and of all the relatives present, c is the value that gives the highest function value.

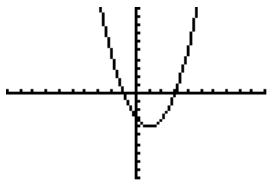


The precise definitions of relative maxima and relative minima are in the glossary of terms for this lesson. These definitions are new to students, so it is worth going over them at the end of the lesson. The main thing to point out to your students is that a relative maximum or minimum actually refers to the x -value, not the ordered pair on the graph of the function or the value of the function at the relative maximum or minimum. The y -value is called the relative maximum (or minimum) value of the function.

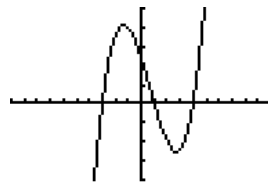
Discussion

For any particular polynomial, can we determine how many relative maxima or minima there are? Consider the following polynomial functions in factored form and their graphs.

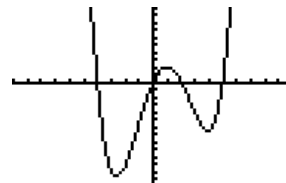
$$f(x) = (x + 1)(x - 3)$$



$$g(x) = (x + 3)(x - 1)(x - 4)$$



$$h(x) = (x)(x + 4)(x - 2)(x - 5)$$



Degree of each polynomial:

2

3

4

Number of x -intercepts in each graph:

2

3

4

Number of relative maxima or minima in each graph:

1

2

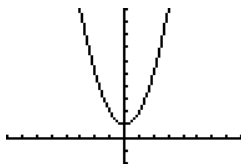
3

What observations can we make from this information?

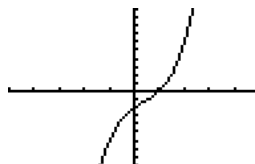
The number of relative maxima and minima is one less than the degree and one less than the number of x -intercepts.

Is this true for every polynomial? Consider the examples below.

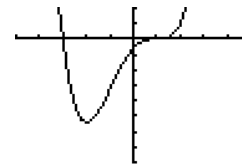
$$r(x) = x^2 + 1$$



$$s(x) = (x^2 + 2)(x - 1)$$



$$t(x) = (x + 3)(x - 1)(x - 1)(x - 1)$$



Degree of each polynomial:

2

3

4

Number of x -intercepts in each graph:

0

1

2

Number of relative maxima or minima in each graph:

1

0

1

What observations can we make from this information?

The observations made in the previous examples do not hold for these examples, so it is difficult to determine from the degree of the polynomial function the number of relative maxima or minima in the graph of the function. What we can say is that for a degree n polynomial function, there are at most $n - 1$ relative maxima or relative minima. You can also think about the information you can get from a graph. If a graph of a polynomial function has n relative maxima or minima, you can say that the degree of the polynomial is at least $n + 1$.

Closing (1 minute)

- By looking at the factored form of a polynomial, we can identify important characteristics of the graph such as x -intercepts and relative maxima and minima of the function, which in turn allow us to develop a sketch of the graph.
- A polynomial of degree n may have up to n x -intercepts.
- A polynomial of degree n may have up to $n - 1$ relative maxima and minima.

Relevant Vocabulary

Increasing/Decreasing: Given a function f whose domain and range are subsets of the real numbers and I is an interval contained within the domain, the function is called *increasing on the interval I* if

$$f(x_1) < f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I.$$

It is called *decreasing on the interval I* if

$$f(x_1) > f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I.$$

Relative Maximum: Let f be a function whose domain and range are subsets of the real numbers. The function has a *relative maximum at c* if there exists an open interval I of the domain that contains c such that

$$f(x) \leq f(c) \text{ for all } x \text{ in the interval } I.$$

If c is a relative maximum, then the value $f(c)$ is called the *relative maximum value*.

Relative Minimum: Let f be a function whose domain and range are subsets of the real numbers. The function has a *relative minimum at c* if there exists an open interval I of the domain that contains c such that

$$f(x) \geq f(c) \text{ for all } x \text{ in the interval } I.$$

If c is a relative minimum, then the value $f(c)$ is called the *relative minimum value*.

Graph of f : Given a function f whose domain D and the range are subsets of the real numbers, the graph of f is the set of ordered pairs in the Cartesian plane given by

$$\{(x, f(x)) \mid x \in D\}.$$

Graph of $y = f(x)$: Given a function f whose domain D and the range are subsets of the real numbers, the *graph of $y = f(x)$* is the set of ordered pairs (x, y) in the Cartesian plane given by

$$\{(x, y) \mid x \in D \text{ and } y = f(x)\}.$$

Lesson Summary

A polynomial of degree n may have up to n x -intercepts and up to $n - 1$ relative maximum/minimum points.

A relative maximum is the x -value c that produces the highest point on a graph of f in a circle around $(c, f(c))$. That highest value $f(c)$ is a relative maximum value.

A relative minimum is the x -value d that produces the lowest point on a graph of f in a circle around $(d, f(d))$. That lowest value $f(d)$ is a relative minimum value.

Exit Ticket (5 minutes)

Name _____

Date _____

Lesson 14: Graphing Factored Polynomials

Exit Ticket

Sketch a graph of the function $f(x) = x^3 + x^2 - 4x - 4$ on the interval $[-2, 2]$ by finding the zeros and determining the sign of the function between zeros. Explain how the structure of the equation helps guide your sketch.

Exit Ticket Sample Solutions

Sketch a graph of the function $f(x) = x^3 + x^2 - 4x - 4$ by finding the zeros and determining the sign of the function between zeros. Explain how the structure of the equation helps guide your sketch.

$$f(x) = (x + 1)(x + 2)(x - 2)$$

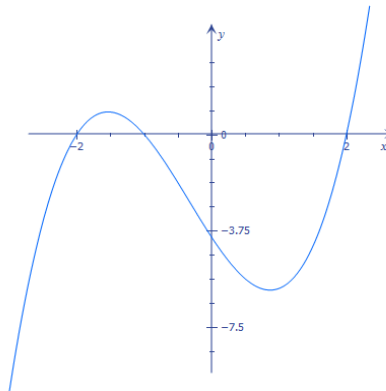
Zeros: $-1, -2, 2$

For $x < -2$: $f(-3) = -10$, so the graph is below the x -axis on this interval.

For $-2 < x < -1$: $f(-1.5) = 0.875$, so the graph is above the x -axis on this interval.

For $-1 < x < 2$: $f(0) = -4$, so the graph is below the x -axis on this interval.

For $x > 2$: $f(3) = 20$, so the graph is above the x -axis on this interval.

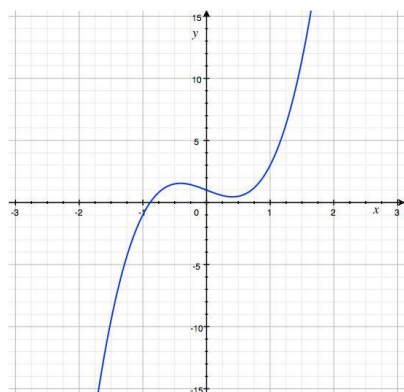


Problem Set Sample Solutions

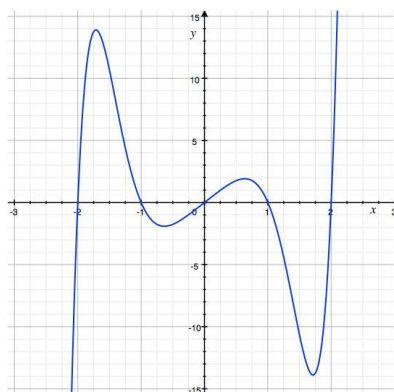
- For each function below, identify the largest possible number of x -intercepts and the largest possible number of relative maximum and minimum points based on the degree of the polynomial. Then use a calculator or graphing utility to graph the function and find the actual number of x -intercepts and relative maximum/minimum points.

- $f(x) = 4x^3 - 2x + 1$
- $g(x) = x^7 - 4x^5 - x^3 + 4x$
- $h(x) = x^4 + 4x^3 + 2x^2 - 4x + 2$

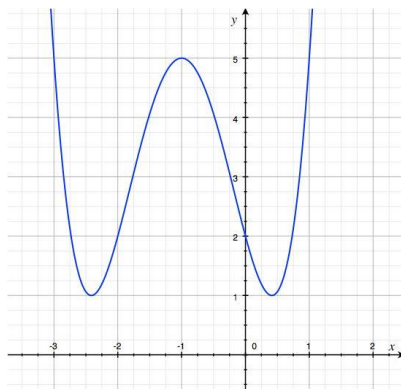
Function	Largest number of x -intercepts	Largest number of relative max/min	Actual number of x -intercepts	Actual number of relative max/min
a. $f(x)$	3	2	1	2
b. $g(x)$	7	6	5	4
c. $h(x)$	4	3	0	3



a. $f(x) = 4x^3 - 2x + 1$



b. $g(x) = x^7 - 4x^5 - x^3 + 4x$



c. $h(x) = x^4 + 4x^3 + 2x^2 - 4x + 2$

2. Sketch a graph of the function $f(x) = \frac{1}{2}(x+5)(x+1)(x-2)$ by finding the zeros and determining the sign of the values of the function between zeros.

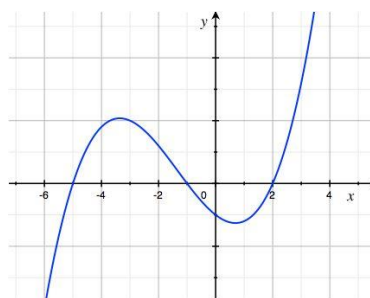
The zeros are -5 , -1 , and 2 .

For $x < -5$: $f(-6) = -20$, so the graph is below the x -axis for $x < -5$.

For $-5 < x < -1$: $f(-3) = 10$, so the graph is above the x -axis for $-5 < x < -1$.

For $-1 < x < 2$: $f(0) = -5$, so the graph is below the x -axis for $-1 < x < 2$.

For $x > 2$: $f(3) = 16$, so the graph is above the x -axis for $x > 2$.



3. Sketch a graph of the function $f(x) = -(x+2)(x-4)(x-6)$ by finding the zeros and determining the sign of the values of the function between zeros.

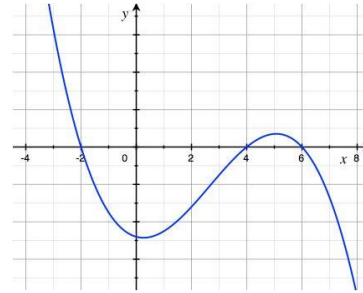
The zeros are -2 , 4 , and 6 .

For $x < -2$: $f(-3) = 63$, so the graph is above the x -axis for $x < -2$.

For $-2 < x < 4$: $f(0) = -48$, so the graph is below the x -axis for $-2 < x < 4$.

For $4 < x < 6$: $f(5) = 7$, so the graph is above the x -axis for $4 < x < 6$.

For $x > 6$: $f(7) = -27$, so the graph is below the x -axis for $x > 6$.



4. Sketch a graph of the function $f(x) = x^3 - 2x^2 - x + 2$ by finding the zeros and determining the sign of the values of the function between zeros.

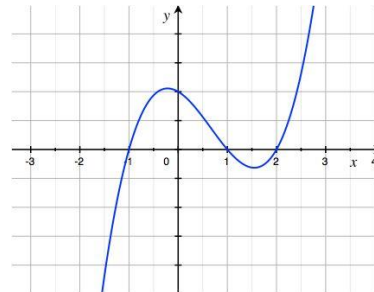
We can factor by grouping to find $f(x) = (x^2 - 1)(x - 2)$. The zeros are -1 , 1 , and 2 .

For $x < -1$: $f(-2) = -12$, so the graph is below the x -axis for $x < -1$.

For $-1 < x < 1$: $f(0) = 2$, so the graph is above the x -axis for $-1 < x < 1$.

For $1 < x < 2$: $f\left(\frac{3}{2}\right) = -\frac{5}{8}$, so the graph is below the x -axis for $1 < x < 2$.

For $x > 2$: $f(3) = 8$, so the graph is above the x -axis for $x > 2$.



5. Sketch a graph of the function $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ by determining the sign of the values of the function between the zeros -1 , 1 , and 3 .

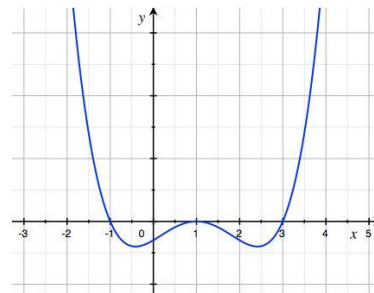
We are told that the zeros are -1 , 1 , and 3 .

For $x < -1$: $f(-2) = 45$, so the graph is above the x -axis for $x < -1$.

For $-1 < x < 1$: $f(0) = -3$, so the graph is below the x -axis for $-1 < x < 1$.

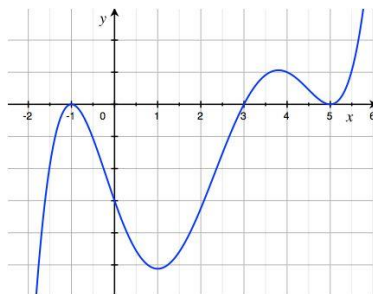
For $1 < x < 3$: $f(2) = -3$, so the graph is below the x -axis for $1 < x < 3$.

For $x > 3$: $f(4) = 45$, so the graph is above the x -axis for $x > 3$.



6. A function f has zeros at -1 , 3 , and 5 . We know that $f(-2)$ and $f(2)$ are negative, while $f(4)$ and $f(6)$ are positive. Sketch a graph of f .

From the information given, the graph of f lies below the x -axis for $x < -1$ and $-1 < x < 3$ and that it touches the x -axis at -1 . Similarly, we know that the graph of f lies above the x -axis for $3 < x < 5$ and $5 < x$ and that it touches the x -axis at 5 . We also know that the graph crosses the x -axis at 3 .



7. The function $h(t) = -16t^2 + 33t + 45$ represents the height of a ball tossed upward from the roof of a building 45 feet in the air after t seconds. Without graphing, determine when the ball will hit the ground.

Factor: $h(t) = (t - 3)(-16t - 15)$

Solve $h(t) = 0$: $(t - 3)(-16t - 15) = 0$

$t = 3$ seconds or $t = -\frac{15}{16}$ seconds.

The ball hits the ground at time 3 seconds; the solution $-\frac{15}{16}$ does not make sense in the context of the problem.