

Lesson 12: Overcoming Obstacles in Factoring

Classwork

Example 1

Find all real solutions to the equation $(x^2 - 6x + 3)(2x^2 - 4x - 7) = 0$.

Exercise 1

Factor and find all real solutions to the equation $(x^2 - 2x - 4)(3x^2 + 8x - 3) = 0$.

Example 2

Find all solutions to $x^3 + 3x^2 - 9x - 27 = 0$ by factoring the equation.

Exercise 2

Find all real solutions to $x^3 - 5x^2 - 4x + 20 = 0$.

Exercise 3

Find all real solutions to $x^3 - 8x^2 - 2x + 16 = 0$.

Lesson Summary

In this lesson, we learned some techniques to use when faced with factoring polynomials and solving polynomial equations.

- If a fourth degree polynomial can be factored into two quadratic expressions, then each quadratic expression might be factorable either using the quadratic formula or by completing the square.
- Some third degree polynomials can be factored using the technique of factoring by grouping.

Problem Set

1. Solve each of the following equations by completing the square.

- $x^2 - 6x + 2 = 0$
- $x^2 - 4x = -1$
- $x^2 + x - \frac{3}{4} = 0$
- $3x^2 - 9x = -6$
- $(2x^2 - 5x + 2)(3x^2 - 4x + 1) = 0$
- $x^4 - 4x^2 + 2 = 0$

2. Solve each of the following equations using the quadratic formula.

- $x^2 - 5x - 3 = 0$
- $(6x^2 - 7x + 2)(x^2 - 5x + 5) = 0$
- $(3x^2 - 13x + 14)(x^2 - 4x + 1) = 0$

3. Not all of the expressions in the equations below can be factored using the techniques discussed so far in this course. First, determine if the expression can be factored with real coefficients. If so, factor the expression and find all real solutions to the equation.

- $x^2 - 5x - 24 = 0$
- $3x^2 + 5x - 2 = 0$
- $x^2 + 2x + 4 = 0$
- $x^3 + 3x^2 - 2x + 6 = 0$
- $x^3 + 3x^2 + 2x + 6 = 0$
- $2x^3 + x^2 - 6x - 3 = 0$
- $8x^3 - 12x^2 + 2x - 3 = 0$
- $6x^3 + 8x^2 + 15x + 20 = 0$
- $4x^3 + 2x^2 - 36x - 18 = 0$
- $x^2 - \frac{1}{2}x - \frac{15}{2} = 0$

4. Solve the following equations by bringing all terms to one side of the equation and factoring out the greatest common factor.
- a. $(x - 2)(x - 1) = (x - 2)(x + 1)$
 - b. $(2x + 3)(x - 4) = (2x + 3)(x + 5)$
 - c. $(x - 1)(2x + 3) = (x - 1)(x + 2)$
 - d. $(x^2 + 1)(3x - 7) = (x^2 + 1)(3x + 2)$
 - e. $(x + 3)(2x^2 + 7) = (x + 3)(x^2 + 8)$
5. Consider the expression $x^4 + 1$. Since $x^2 + 1$ does not factor with real number coefficients, we might expect that $x^4 + 1$ also does not factor with real number coefficients. In this exercise, we will investigate the possibility of factoring $x^4 + 1$.
- a. Simplify the expression $(x^2 + 1)^2 - 2x^2$.
 - b. Factor $(x^2 + 1)^2 - 2x^2$ as a difference of squares.
 - c. Is it possible to factor $x^4 + 1$ with real number coefficients? Explain