## **Mathematics Curriculum**

## Topic B: Factoring—Its Use and Its Obstacles

## N-Q.A.2, A-SSE.A.2, A-APR.B.2, A-APR.B.3, A-APR.D.6, F-IF.C.7c

Focus Standard:	N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. $\star$
	A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
	A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
	A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
	A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
	F-IF.C.7	<ul> <li>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</li> <li>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> </ul>
Instructional Days:	10	
Lesson 12:	Overcoming Obstacles in Factoring (S) <sup>1</sup>	
Lesson 13:	Mastering Factoring (P)	
Lesson 14:	Graphing Factored Polynomials (S)	
Lesson 15:	Structure in Graphs of Polynomial Functions (P)	
Lessons 16–17:	Modeling with Polynomials—An Introduction (M, M)	
Lesson 18:	Overcoming a Second Obstacle in Factoring—What If There Is a Remainder? (P)	

<sup>&</sup>lt;sup>1</sup> Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson



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Lesson 19: The Remainder Theorem (S) Lessons 20–21: Modeling Riverbeds with Polynomials (M, M)

Armed with a newfound knowledge of the value of factoring, students develop their facility with factoring and then apply the benefits to graphing polynomial equations in Topic B. In Lessons 12–13, students are presented with the first obstacle to solving equations successfully. While dividing a polynomial by a given factor to find a missing factor is easily accessible, factoring without knowing one of the factors is challenging. Students recall the work with factoring done in Algebra I and expand on it to master factoring polynomials with degree greater than two, emphasizing the technique of factoring by grouping.

In Lessons 14–15, students find that another advantage to rewriting polynomial expressions in factored form is how easily a polynomial function written in this form can be graphed. Students read word problems to answer polynomial questions by examining key features of their graphs. They notice the relationship between the number of times a factor is repeated and the behavior of the graph at that zero (i.e., when a factor is repeated an even number of times, the graph of the polynomial will touch the *x*-axis and "bounce" back off, whereas when a factor occurs only once or an odd number of times, the graph of the polynomial at that zero will "cut through" the *x*-axis). In these lessons, students will compare hand plots to graphing-calculator plots and zoom in on the graph to examine its features more closely.

In Lessons 16–17, students encounter a series of more serious modeling questions associated with polynomials, developing their fluency in translating between verbal, numeric, algebraic, and graphical thinking. One example of the modeling questions posed in this lesson is how to find the maximum possible volume of a box created from a flat piece of cardboard with fixed dimensions.

In Lessons 18–19, students are presented with their second obstacle: "What if there is a remainder?" They learn the Remainder Theorem and apply it to further understand the connection between the factors and zeros of a polynomial and how this relates to the graph of a polynomial function. Students explore how to determine the smallest possible degree for a depicted polynomial and how information such as the value of the *y*-intercept will be reflected in the equation of the polynomial.

The topic culminates with two modeling lessons (Lessons 20–21) involving approximating the area of the cross-section of a riverbed to model the volume of flow. The problem description includes a graph of a polynomial equation that could be used to model the situation, and students are challenged to find the polynomial equation itself.



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