Lesson 11: The Special Role of Zero in Factoring

Classwork

Opening Exercise

Find all solutions to the equation .

Exercise 1

1. Find the solutions of .

**Example 1**

Suppose we know that the polynomial equation has three real solutions and that one of the factors of is . How can we find all three solutions to the given equation?

Exercises 2–5

1. Find the zeros of the following polynomial functions, with their multiplicities.
2. Find a polynomial function that has the following zeros and multiplicities. What is the degree of your polynomial?

|  |  |
| --- | --- |
| Zero | Multiplicity |
|  |  |
|  |  |
|  |  |
|  |  |

1. Is there more than one polynomial function that has the same zeros and multiplicities as the one you found in Exercise 3?
2. Can you find a rule that relates the multiplicities of the zeros to the degree of the polynomial function?

Relevant Vocabulary Terms

In the definitions below, the symbol stands for the set of real numbers.

**Function:**  A *function* is a correspondence between two sets, and *,* in which each element of is assigned to one and only one element of .

The set in the definition above is called the *domain of the function.* The *range (or image)* of the function is the subset of , denoted , that is defined by the following property: is an element of if and only if there is an in such that .

If where can be any real number, then the domain is all real numbers (denoted ), and the range is the set of non-negative real numbers.

**Polynomial Function:**  Given a polynomial expression in one variable, a *polynomial function in one variable* is a function such that for each real number in the domain, is the value found by substituting the number into all instances of the variable symbol in the polynomial expression and evaluating.

It can be shown that if a function is a polynomial function, then there is some non-negative integer and collection of real numbers , , ,, with such that the function satisfies the equation

for every real number in the domain, which is called the *standard form of the polynomial function.* The function   
, where can be any real number, is an example of a function written in standard form.

**Degree of a Polynomial Function:** The *degree of a polynomial function* is the degree of the polynomial expression used to define the polynomial function.

The degree of is 3, but the degree of is because when is put into standard form, it is .

**Constant Function:** A *constant function* is a polynomial function of degree 0. A constant function is of the form , for a constant .

**Linear Function:**  A *linear function* is a polynomial function of degree . A linear function is of the form , for constants and with .

**Quadratic Function:** A *quadratic function* is a polynomial function of degree. A quadratic function is in *standard form* if it is written in the form , for constants , , with and any real number.

**Cubic Function:**  A *cubic function* is a polynomial function of degree . A cubic function is of the form for constants with .

**Zeros or Roots of a Function:**  A *zero* (or *root*) of a function is a number of the domain such that   
. A zero of a function is an element in the solution set of the equation .

Problem Set

Lesson Summary

Given any two polynomial functions and , the solution set of the equation can be quickly found by solving the two equations and and combining the solutions into one set.

The number is zero of a polynomial function with multiplicity if the factored form of contains .

For Problems 1–4, find all solutions to the given equations.

2. Find four solutions to the equation .
3. Find the zeros with multiplicity for the function
4. Find two different polynomial functions that have zeros at , , and of multiplicity .
5. Find two different polynomial functions that have a zero at of multiplicity and a zero at of multiplicity .
6. Find three solutions to the equation .
7. Find two solutions to the equation .
8. If ,,, are non-zero numbers, find the solutions to the equation in terms of , ,,.

Use the identity to solve the equations given in Problems 12–13.

1. Consider the polynomial function .
   1. Divide by the divisor and rewrite in the form .
   2. Evaluate .
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   1. Divide by the divisor and rewrite in the form .
   2. Evaluate .
3. Consider the polynomial function .
   1. Divide by the divisor and rewrite in the form .
   2. Evaluate .
4. Consider the polynomial function .
   1. Divide by the divisor and rewrite in the form .
   2. Evaluate .
5. Make a conjecture based on the results of Questions 14—17.