Lesson 7: Mental Math

Classwork

Opening Exercise

a. How are these two equations related?

$$\frac{x^2-1}{x+1} = x-1 \quad \text{and} \quad x^2-1 = (x+1)(x-1)$$

b. Explain the relationship between the polynomial identities $x^2 - 1 = (x+1)(x-1)$ and $x^2 - a^2 = (x-a)(x+a)$.

Exercises 1–3

1. Compute the following products using the identity $x^2 - a^2 = (x-a)(x+a)$. Show your steps.

   a. $6 \cdot 8$

   b. $11 \cdot 19$

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c. $23 \cdot 17$

d. $34 \cdot 26$

2. Find two additional factors of $2^{100} - 1$.

3. Show that $8^3 - 1$ is divisible by 7.

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Lesson Summary

Based on the work in this lesson, we can convert differences of squares into products (and vice versa) using

$$x^2 - a^2 = (x - a)(x + a).$$

If x, a, and n are integers and $n > 1$, then numbers of the form $x^n - a^n$ are not prime because

$$x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-2}x + a^{n-1}).$$

Problem Set

1. Using an appropriate polynomial identity, quickly compute the following products. Show each step. Be sure to state your values for x and a.
   a. $41 \cdot 19$
   b. $993 \cdot 1{,}007$
   c. $213 \cdot 187$
   d. $29 \cdot 51$
   e. $125 \cdot 75$

2. Give the general steps you take to determine x and a when asked to compute a product such as those in Problem 1.

3. Why is $17 \cdot 23$ easier to compute than $17 \cdot 22$?

4. Rewrite the following differences of squares as a product of two integers.
   a. $81 - 1$
   b. $400 - 121$

5. Quickly compute the following differences of squares.
   a. $64^2 - 14^2$
   b. $112^2 - 88^2$
   c. $785^2 - 215^2$

6. Is 323 prime? Use the fact that $18^2 = 324$ and an identity to support your answer.

7. The number $2^3 - 1$ is prime and so are $2^5 - 1$ and $2^7 - 1$. Does that mean $2^9 - 1$ is prime? Explain why or why not.

8. Show that 9,999,999,991 is not prime without using a calculator or computer.

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9. Show that $999{,}973$ is not prime without using a calculator or computer.

10. Find a value of b so that the expression $b^n - 1$ is always divisible by 5 for any positive integer n. Explain why your value of b works for any positive integer n.

11. Find a value of b so that the expression $b^n - 1$ is always divisible by 7 for any positive integer n. Explain why your value of b works for any positive integer n.

12. Find a value of b so that the expression $b^n - 1$ is divisible by both 7 and 9 for any positive integer n. Explain why your value of b works for any positive integer n.

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