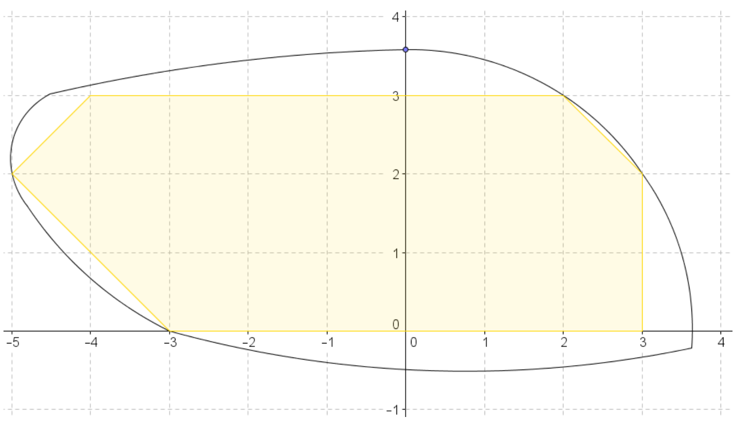
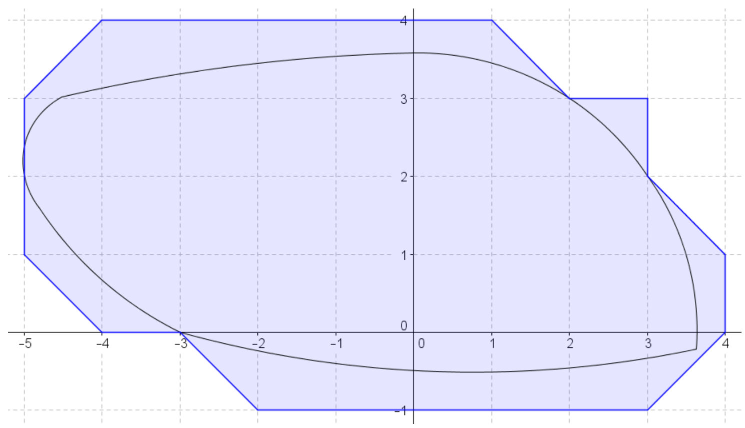
Name ____ Date ____________________

Lesson 1: What Is Area?

Exit Ticket

1. Explain how to use the shaded polygonal regions shown to estimate the area A inside the curve.





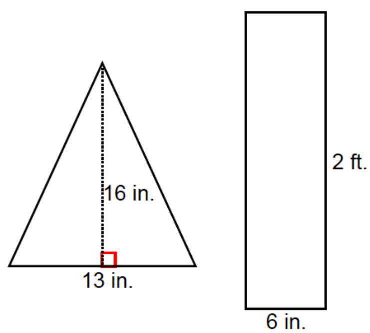
2. Use Problem 1 to find an average estimate for the area inside the curve.

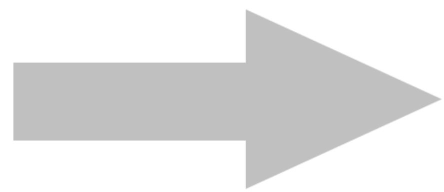
Name ____ Date ____________________

Lesson 2: Properties of Area

Exit Ticket

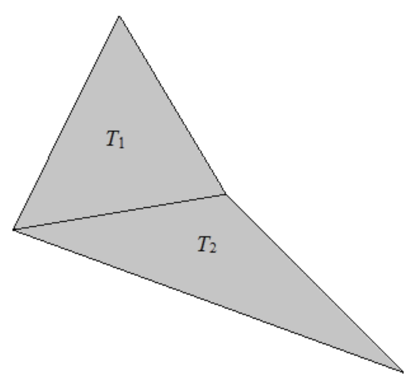
1. Wood pieces in the following shapes and sizes are nailed together in order to create a sign in the shape of an arrow. The pieces are nailed together so that the rectangular piece overlaps with the triangular piece by 4 in. What is the area of the region in the shape of the arrow?





arrow-shaped sign

2. A quadrilateral Q is the union of two triangles T_1 and T_2 that meet along a common side as shown in the diagram. Explain why $\text{Area}(Q) = \text{Area}(T_1) + \text{Area}(T_2)$.

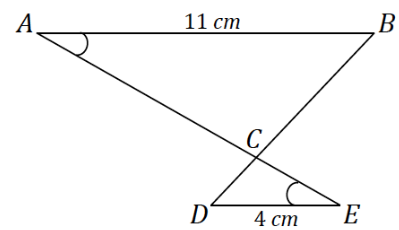


Name ____ Date ____________________

Lesson 3: The Scaling Principle for Area

Exit Ticket

In the following figure, $\overline{AE}$ and $\overline{BD}$ are segments.



a. $\triangle ABC$ and $\triangle CDE$ are similar. How do we know this?

b. What is the scale factor of the similarity transformation that takes $\triangle ABC$ to $\triangle CDE$?

c. What is the value of the ratio of the area of $\triangle ABC$ to the area of $\triangle CDE$? Explain how you know.

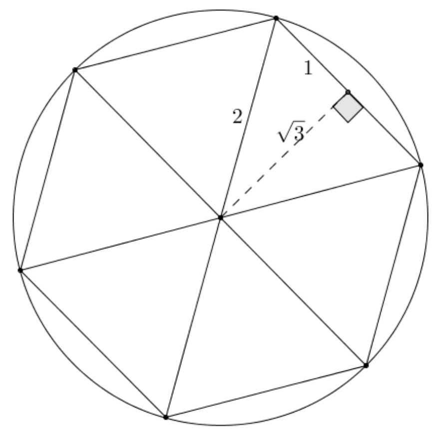
d. If the area of $\triangle ABC$ is 30 cm^2, what is the area of $\triangle CDE$?

Name ____ Date ____________________

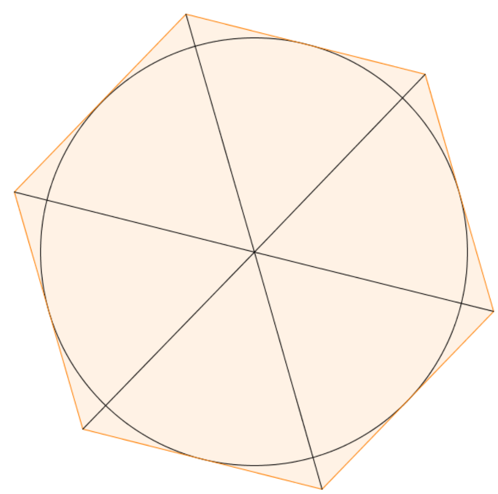
Lesson 4: Proving the Area of a Disk

Exit Ticket

1. Approximate the area of a disk of radius 2 using an inscribed regular hexagon.



2. Approximate the area of a disk of radius 2 using a circumscribed regular hexagon.

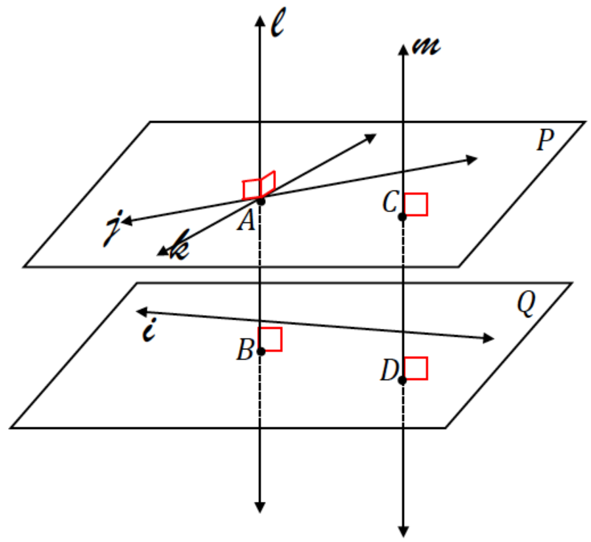


3. Based on the areas of the inscribed and circumscribed hexagons, what is an approximate area of the given disk? What is the area of the disk by the area formula, and how does your approximation compare?

Name ____ Date ____________________

Lesson 5: Three-Dimensional Space

Exit Ticket



1. What can be concluded about the relationship between line ℓ and plane P? Why?

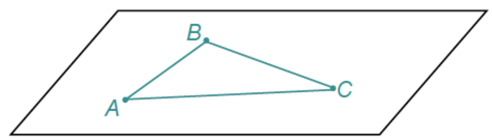
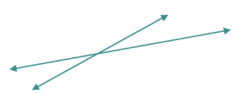
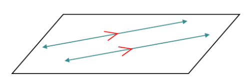
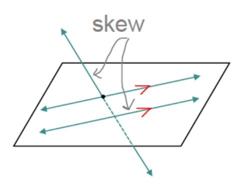
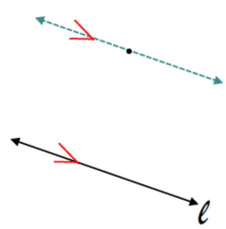
2. What can be concluded about the relationship between planes P and Q? Why?

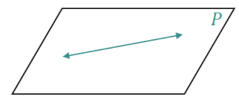
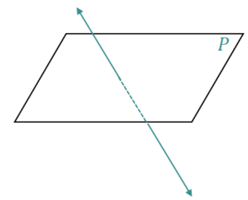
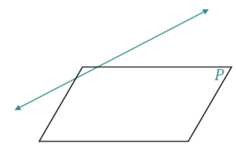
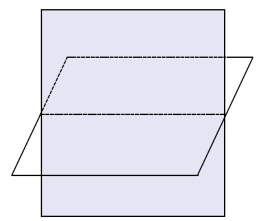
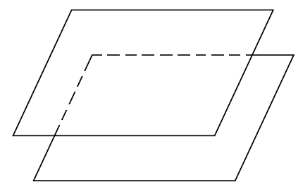
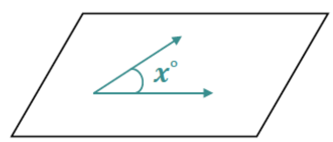
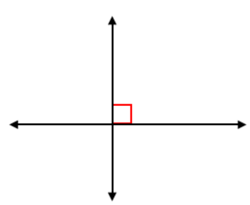
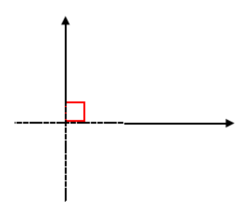
3. What can be concluded about the relationship between lines ℓ and m? Why?

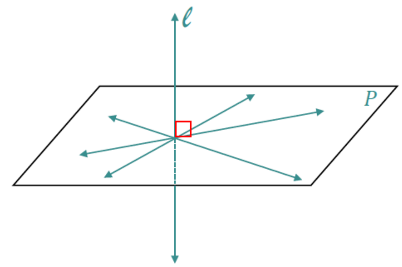
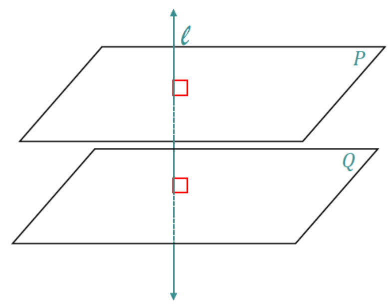
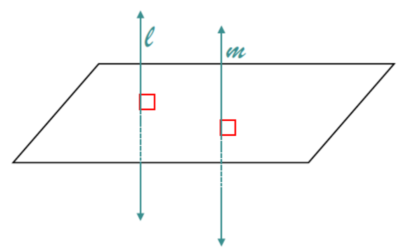
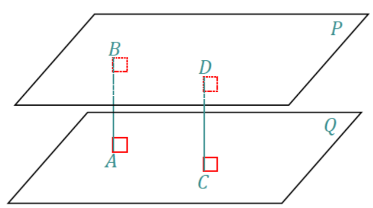
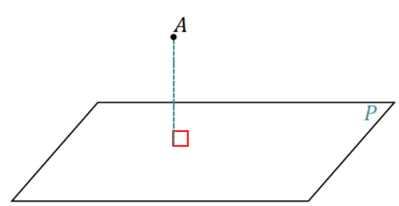
4. What can be concluded about segments $\overline{AB}$ and $\overline{CD}$?

5. Line j lies in plane P, and line i lies in plane Q. What can be concluded about the relationship between lines i and j?

Table 2: Properties of Points, Lines, and Planes in Three-Dimensional Space

	Property	Diagram
1		
2		
3		(a)  (b)  (c) 
4		

5		**(a)** 	**(b)** 	**(c)** 
6		**(a)** 	**(b)** 	
7				
8				

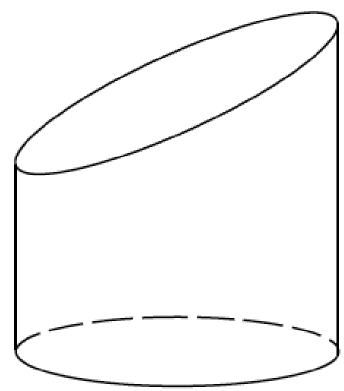
9		
10		
11		
12		 $AB = CD$
13		

Name ____ Date ____________________

Lesson 6: General Prisms and Cylinders and Their Cross-Sections

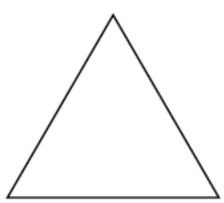
Exit Ticket

1. Is this a cylinder? Explain why or why not.

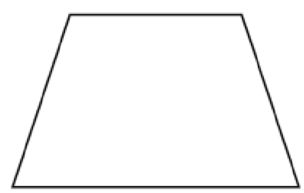


2. For each of the following cross-sections, sketch the figure from which the cross-section was taken.

a.

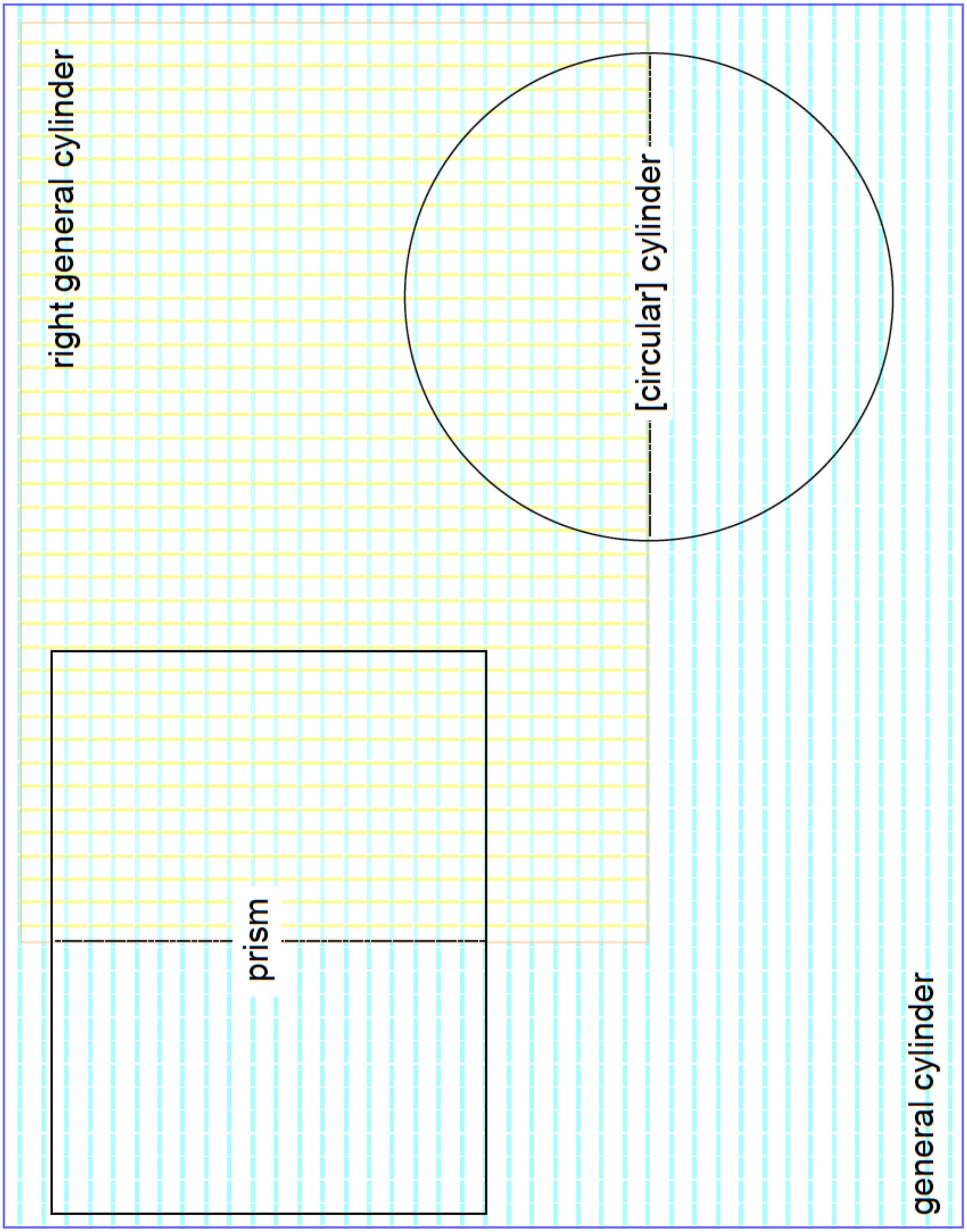


b.



Exploratory Challenge

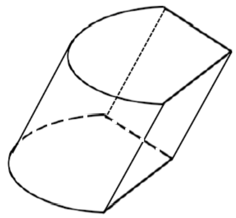
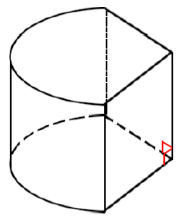
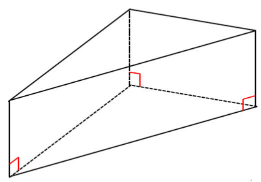
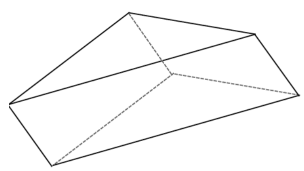
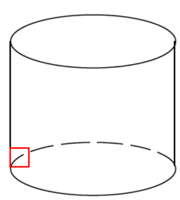
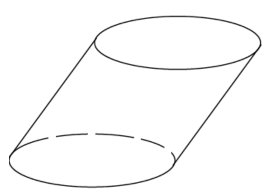
Option 1



Option 2

	Figure and Description	Sketch of Figure	Sketch of Cross-Section
1.	**General Cylinder** Let E and E' be two parallel planes, let B be a region in the plane E, and let L be a line which intersects E and E' but not B. At each point P of B, consider the segment $\overline{PP'}$ parallel to L, joining P to a point P' of the plane E'. The union of all these segments is called a *general cylinder with base* B.		
2.	**Right General Cylinder** A general cylinder whose lateral edges are perpendicular to the bases.		
3.	**Right Prism** A general cylinder whose lateral edges are perpendicular to a polygonal base.		
4.	**Oblique Prism** A general cylinder whose lateral edges are not perpendicular to a polygonal base.		
5.	**Right Cylinder** A general cylinder whose lateral edges are perpendicular to a circular base.		
6.	**Oblique Cylinder** A general cylinder whose lateral edges are not perpendicular to a circular base.		

Option 3

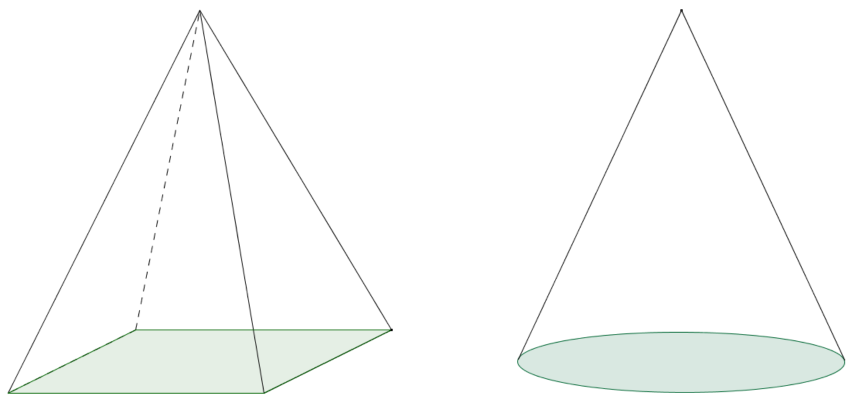
	Figure and Description	Sketch of Figure	Sketch of Cross-Section
1.	**General Cylinder**		
2.	**Right General Cylinder**		
3.	**Right Prism**		
4.	**Oblique Prism**		
5.	**Right Cylinder**		
6.	**Oblique Cylinder**		

Name ____ Date ____________________

Lesson 7: General Pyramids and Cones and Their Cross-Sections

Exit Ticket

The diagram below shows a circular cone and a general pyramid. The bases of the cones are equal in area, and the solids have equal heights.



a. Sketch a slice in each cone that is parallel to the base of the cone and $\frac{2}{3}$ closer to the vertex than the base plane.

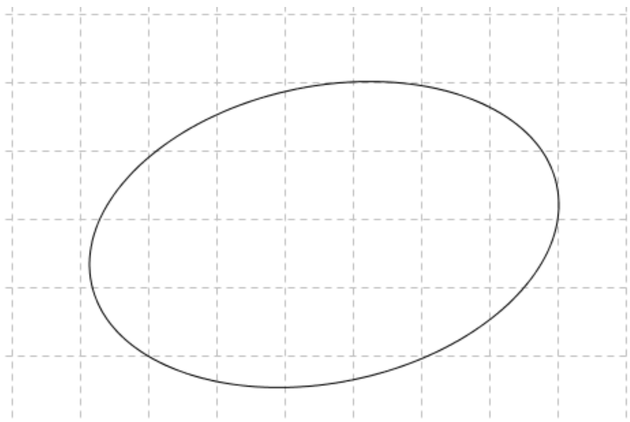
b. If the area of the base of the circular cone is 616 units^2, find the exact area of the slice drawn in the pyramid.

Name ____ Date ____________________

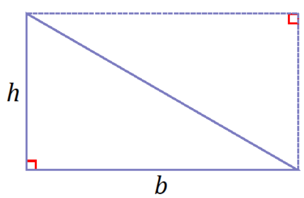
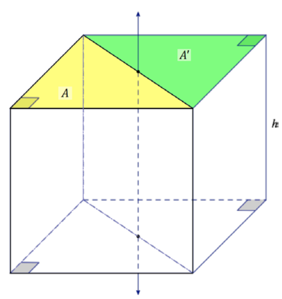
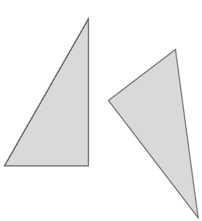
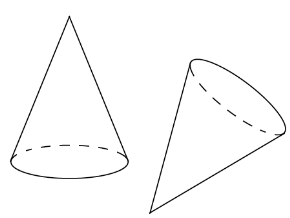
Lesson 8: Definition and Properties of Volume

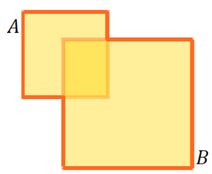
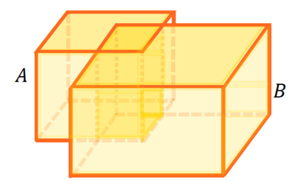
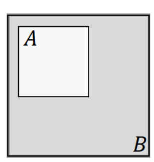
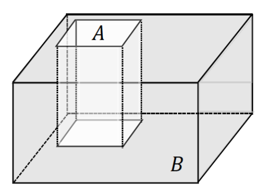
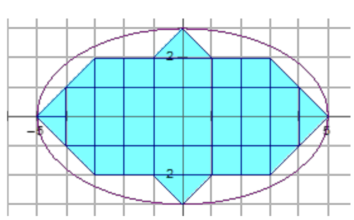
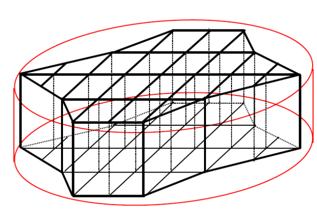
Exit Ticket

The diagram shows the base of a cylinder. The height of the cylinder is 14 cm. If each square in the grid is $1 \text{ cm} \times 1 \text{ cm}$, make an approximation of the volume of the cylinder. Explain your reasoning.



Opening

Area Properties	Volume Properties
1. The area of a set in two dimensions is a number greater than or equal to zero that measures the size of the set and not the shape.	1.
2. The area of a rectangle is given by the formula length $\times$ width. The area of a triangle is given by the formula $\frac{1}{2} \times$ base $\times$ height. A polygonal region is the union of finitely many non-overlapping triangular regions and has area the sum of the areas of the triangles. 	2. 
3. Congruent regions have the same area. 	3. Congruent solids have the same volume. 

4. The area of the union of two regions is the sum of the areas minus the area of the intersection: $\text{Area}(A \cup B) = \text{Area}(A) + \text{Area}(B) - \text{Area}(A \cap B)$ 	4. 
5. The area of the difference of two regions where one is contained in the other is the difference of the areas: If $A \subseteq B$, then $\text{Area}(B - A) = \text{Area}(B) - \text{Area}(A)$. 	5. 
6. The area a of a region A can be estimated by using polygonal regions S and T so that S is contained in A and A is contained in T. Then $\text{Area}(S) \leq a \leq \text{Area}(T)$. 	6. 

Name ____ Date ____________________

Lesson 9: Scaling Principle for Volumes

Exit Ticket

1. Two circular cylinders are similar. The ratio of the areas of their bases is $9:4$. Find the ratio of the volumes of the similar solids.

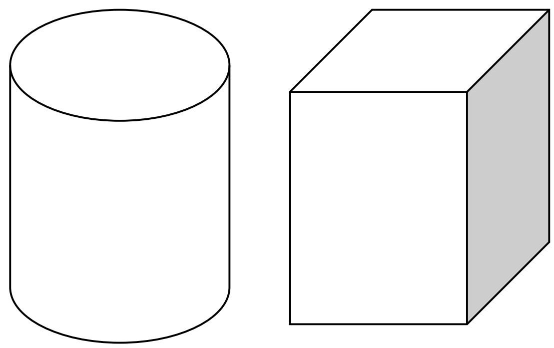
2. The volume of a rectangular pyramid is 60. The width of the base is then scaled by a factor of 3, the length of the base is scaled by a factor of $\frac{5}{2}$, and the height of the pyramid is scaled such that the resulting image has the same volume as the original pyramid. Find the scale factor used for the height of the pyramid.

Name ____ Date ____________________

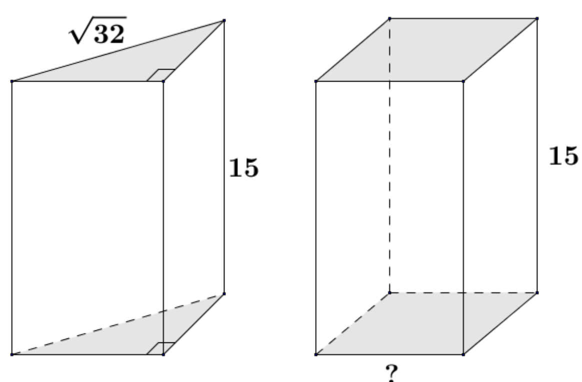
Lesson 10: The Volume of Prisms and Cylinders and Cavalieri's Principle

Exit Ticket

1. Morgan tells you that Cavalieri's principle cannot apply to the cylinders shown below because their bases are different. Do you agree or disagree? Explain.



2. A triangular prism has an isosceles right triangular base with a hypotenuse of $\sqrt{32}$ and a prism height of 15. A square prism has a height of 15 and its volume is equal to that of the triangular prism. What are the dimensions of the square base?

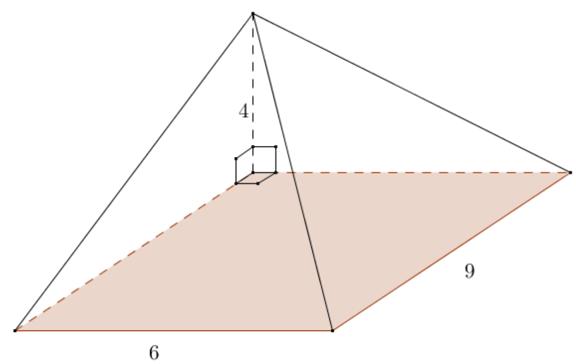


Name ____ Date ____________________

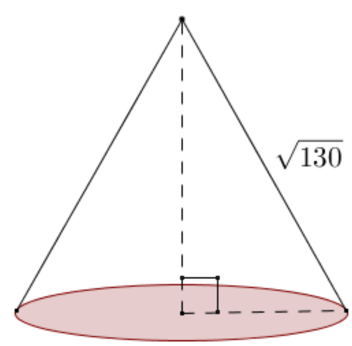
Lesson 11: The Volume Formula of a Pyramid and Cone

Exit Ticket

1. Find the volume of the rectangular pyramid shown.



2. The right circular cone shown has a base with radius of 7. The slant height of the cone's lateral surface is $\sqrt{130}$. Find the volume of the cone.

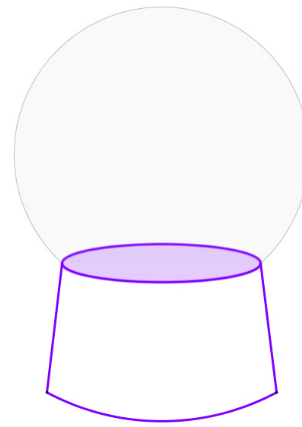


Name ____ Date ____________________

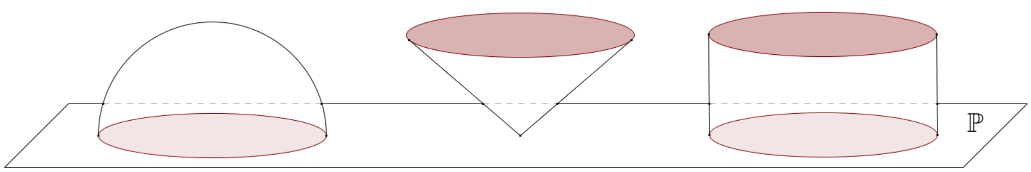
Lesson 12: The Volume Formula of a Sphere

Exit Ticket

1. Snow globes consist of a glass sphere that is filled with liquid and other contents. If the inside radius of the snow globe is 3 in., find the approximate volume of material in cubic inches that can fit inside.



2. The diagram shows a hemisphere, a circular cone, and a circular cylinder with heights and radii both equal to 9.



   a. Sketch parallel cross-sections of each solid at height 3 above plane P.

   b. The base of the hemisphere, the vertex of the cone, and the base of the cylinder lie in base plane P. Sketch parallel cross-sectional disks of the figures at a distance h from the base plane, and then describe how the areas of the cross-sections are related.

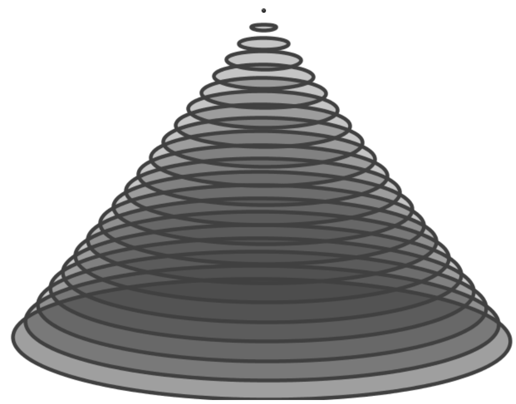
Name ____ Date ____________________

Lesson 13: How Do 3D Printers Work?

Exit Ticket

Lamar is using a 3D printer to construct a circular cone that has a base with radius 6 in.

a. If his 3D printer prints in layers that are 0.004 in. thick (similar to what is shown in the image below), what should be the change in radius for each layer in order to construct a cone with height 4 in.?



b. What is the area of the base of the 27^{th} layer?

c. Approximately how much printing material is required to produce the cone?

Name ____ Date ______________________

1.

a. State the volume formula for a cylinder. Explain why the volume formula works.

b. The volume formula for a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height of the solid. Explain where the $\frac{1}{3}$ comes from in the formula.

c. Give an explanation of how to use the volume formula of a pyramid to show that the volume formula of a circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height of the cone.

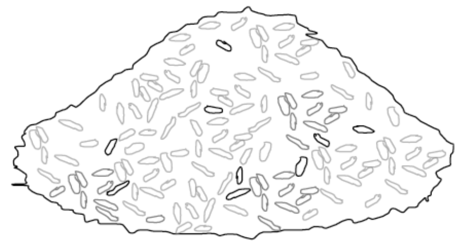
2. A circular cylinder has a radius between 5.50 and 6.00 centimeters and a volume of 225 cubic centimeters. Write an inequality that represents the range of possible heights the cylinder can have to meet this criterion to the nearest hundredth of a centimeter.

3. A machine part is manufactured from a block of iron with circular cylindrical slots. The block of iron has a width of 14 in., a height of 16 in., and a length of 20 in. The number of cylinders drilled out of the block is determined by the weight of the leftover block, which must be less than $1{,}000$ lb.

   a. If iron has a weight of roughly $491\ \text{lb}/\text{ft}^3$, how many cylinders with the same height as the block and with radius 2 in. must be drilled out of the block in order for the remaining solid to weigh less than $1{,}000$ lb.?

   b. If iron ore costs $\$115$ per ton ($1\ \text{ton} = 2200\ \text{lb.}$) and the price of each part is based solely on its weight of iron, how many parts can be purchased with $\$1{,}500$? Explain your answer.

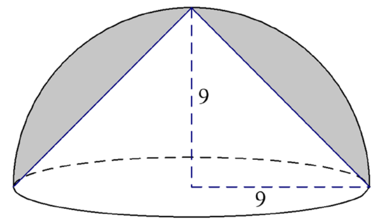
4. Rice falling from an open bag piles up into a figure conical in shape with an approximate radius of 5 cm.



   a. If the angle formed by the slant of the pile with the base is roughly $30°$, write an expression that represents the volume of rice in the pile.

   b. If there are approximately 20 grains of rice in a cubic centimeter, approximately how many grains of rice are in a 4.5-kilogram bag of rice?

5. In a solid hemisphere, a cone is removed as shown. Calculate the volume of the resulting solid. In addition to your solution, provide an explanation of the strategy you used in your solution.



6. Describe the shape of the cross-section of each of the following objects.

Right circular cone:

a. Cut by a plane through the vertex and perpendicular to the base

Square pyramid:

b. Cut by a plane through the vertex and perpendicular to the base

c. Cut by a vertical plane that is parallel to an edge of the base but not passing through the vertex

Sphere with radius r:

d. Describe the radius of the circular cross-section created by a plane through the center of the sphere.

e. Describe the radius of the circular cross-section cut by a plane that does not pass through the center of the sphere.

Triangular Prism:

f. Cut by a plane parallel to a base

g. Cut by a plane parallel to a face

7.

a. A 3×5 rectangle is revolved about one of its sides of length 5 to create a solid of revolution. Find the volume of the solid.

b. A 3-4-5 right triangle is revolved about a leg of length 4 to create a solid of revolution. Describe the solid.

c. A 3-4-5 right triangle is revolved about its legs to create two solids. Find each of the volumes of the two solids created.

d. Show that the volume of the solid created by revolving a 3-4-5 triangle about its hypotenuse is $\frac{48}{5}\pi$.