Topic E

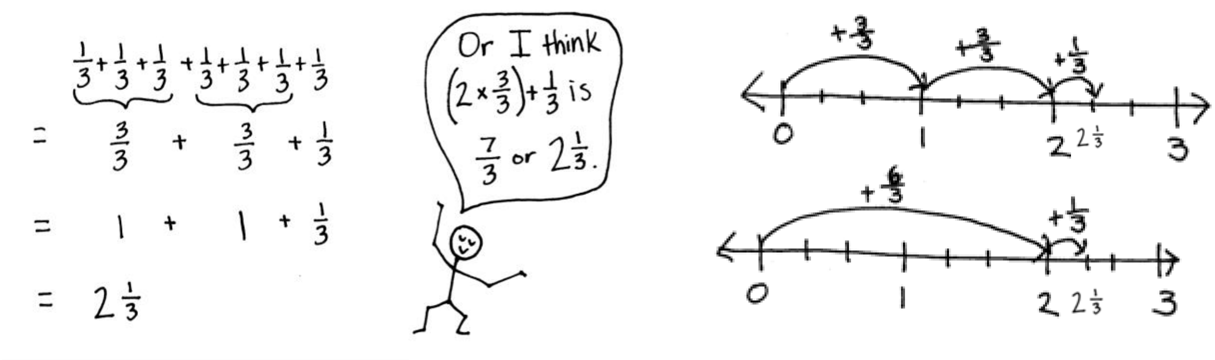
Extending Fraction Equivalence to Fractions Greater Than 1

4.NF.2, 4.NF.3, 4.MD.4, 4.NBT.6, 4.NF.1, 4.NF.4a

Focus Standard:	4.NF.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
	4.NF.3	Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
	4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Instructional Days:	7	
Coherence -Links from:	G3–M5	Fractions as Numbers on the Number Line
-Links to:	G5–M3	Addition and Subtraction of Fractions
	G5–M4	Multiplication and Division of Fractions and Decimal Fractions

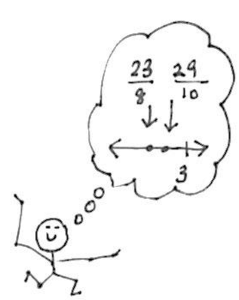
In Topic E, students study equivalence involving both ones and fractional units. In Lesson 22, they use decomposition and visual models to add and subtract fractions less than 1 to and from whole numbers, e.g., $4 + \frac{3}{4} = 4\frac{3}{4}$ and $4 - \frac{3}{4} = (3 + 1) - \frac{3}{4}$, subtracting the fraction from 1 using a number bond and a number line.

Lesson 23 has students use addition and multiplication to build fractions greater than 1 and then represent them on the number line. Fractions can be expressed both in mixed units of a whole number and a fraction or simply as a fraction, as pictured below, e.g., $7 \times \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = \frac{7}{3} = 2\frac{1}{3}$.



In Lessons 24 and 25, students use decompositions to reason about the various equivalent forms in which a fraction greater than or equal to 1 may be presented, both as fractions and as mixed numbers. In Lesson 24, they decompose, for example, 11 fourths into 8 fourths and 3 fourths, $\frac{11}{4} = \frac{8}{4} + \frac{3}{4}$, or they can think of it as $\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = 2 \times \frac{4}{4} + \frac{3}{4} = 2\frac{3}{4}$. In Lesson 25, students are then able to decompose the two wholes into 8 fourths so their original number can then be looked at as $\frac{8}{4} + \frac{3}{4}$ or $\frac{11}{4}$. In this way, they see that $2\frac{3}{4} = \frac{11}{4}$. This fact is further reinforced when they plot $\frac{11}{4}$ on the number line and see that it is at the same point as $2\frac{3}{4}$. Unfortunately, the term *improper fraction* carries some baggage. As many have observed, there is nothing *improper* about an improper fraction. Nevertheless, as a mathematical term, it is useful for describing a particular form in which a fraction may be presented (i.e., a fraction is improper if the numerator is greater than or equal to the denominator). Students do need practice in terms of converting between the various forms a fraction may take, but take care not to foster the misconception that every improper fraction *must* be converted to a mixed number.

Students compare fractions greater than 1 in Lessons 26 and 27. They begin by using their understanding of benchmarks to reason about which of two fractions is greater. This activity builds on students' rounding skills, having them identify the whole numbers and the halfway points between them on the number line. The relationship between the numerator and denominator of a fraction is a key concept here as students consider relationships to whole numbers, e.g., a student might reason that $\frac{23}{8}$ is less than



$\frac{29}{10}$ because $\frac{23}{8}$ is 1 eighth less than 3, but $\frac{29}{10}$ is 1 tenth less than 3. They know each fraction is 1 fractional unit away from 3, and since $\frac{1}{8} > \frac{1}{10}$, then $\frac{23}{8} < \frac{29}{10}$. Students progress to finding and using like denominators to compare and order mixed numbers. Once again, students must use reasoning skills as they determine that, when they have two fractions with the same numerator, the larger fraction will have a larger unit (or smaller denominator). Conversely, when they have two fractions with the same denominator, the larger one will have the larger number of units (or larger numerator).

Lesson 28 concludes the topic with word problems requiring the interpretation of data presented in line plots. Students create line plots to display a given dataset that includes fraction and mixed number values. To do this, they apply their skill in comparing mixed numbers, both through reasoning and the use of common numerators or denominators. For example, a data set might contain both $1\frac{5}{9}$ and $\frac{14}{9}$, giving students the opportunity to determine that they must be plotted at the same point. They also use addition and subtraction to solve the problems.

A Teaching Sequence Toward Mastery of Extending Fraction Equivalence to Fractions Greater Than 1

Objective 1: **Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.**
(Lesson 22)

Objective 2: **Add and multiply unit fractions to build fractions greater than 1 using visual models.**
(Lesson 23)

Objective 3: **Decompose and compose fractions greater than 1 to express them in various forms.**
(Lessons 24–25)

Objective 4: **Compare fractions greater than 1 by reasoning using benchmark fractions.**
(Lesson 26)

Objective 5: **Compare fractions greater than 1 by creating common numerators or denominators.**
(Lesson 27)

Objective 6: **Solve word problems with line plots.**
(Lesson 28)