Counting & Cardinality: A Guide to Kindergarten Mathematics Standards

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Welcome to the UnboundEd Mathematics Guide series! These guides are designed to explain what new, high standards for mathematics say about what students should learn in each grade, and what they mean for curriculum and instruction. This guide, the first for Kindergarten, includes three parts. The first part gives a "tour" of the standards for **Counting & Cardinality** using freely available online resources that you can use or adapt for your class. The second part shows how Counting & Cardinality relate to classification of objects in Kindergarten (**K.MD.B**). And the third part explains where Counting & Cardinality is situated in the progression of learning in Grades K-2. Throughout all of our guides, we include a large number of sample math problems. We strongly suggest tackling these problems yourself to help best understand the methods and strategies we're covering, and the potential challenges your students might face.

Part 1: What do these standards say?

The Counting & Cardinality (CC) domain in Kindergarten focuses on developing meaning for numbers by relating counting to the concept of cardinality—the number of objects in a group. But what does that mean? Children often first learn to count before they understand "how many" objects they are actually counting. For example, children may be able to count to 10 but, given 10 objects to count, they may count "9" or "12" because they are not pairing each number they say with only 1 object. These standards delineate the developmental progression to pairing numbers in the count sequence with objects in a group. ("Because I counted to 3, there are 3 objects here.") This connection is of critical importance of all later work in math.

The standards in the CC domain are foundational skills and understandings that will prepare students for Operations & Algebraic Thinking (OA) and Numbers & Operations in Base Ten (NBT). Counting to tell "how many" are in a group of objects will lead them to understand addition and subtraction (and, later, multiplication and division), as well as understand how to decompose a number from 11-19 into 10 ones and some more ones. For this reason, the K.CC standards are a logical place to begin a year of instruction in Kindergarten.

The Counting & Cardinality domain is composed of three clusters; each cluster has associated standards and is part of the **major work** of the grade, as indicated by the green square above.[1] It's important to spend a large majority of your math instruction teaching the concepts in major clusters. In fact, since K.CC is so fundamental, instruction of and practice with the concepts in these clusters should occur regularly throughout the school year.

The other cluster discussed in this guide comes from the Measurement & Data (MD) domain. ☐ **K.MD.B** is a supporting cluster, and the standard in this cluster can help students deepen their understanding of the standards in the CC domain. Specifically, classifying objects for the purposes of counting these objects supports developing cardinality. We will talk about this connection in detail in Part 2.

Throughout this guide, we'll look at examples of tasks and lessons that focus on students' abilities to look for and make use of structure (**MP.7**). Much of the CC domain is about making sense of the structure of whole numbers, including the notion that adding one results in the next counting number and how this supports understandings of "more" and "less".[2]

> It's important to note that the **clusters, and the standards within the clusters, are not necessarily sequenced in the order in which they have to be taught.** (Standards are only a set of expectations of what students should know and be able to do by the end of each year; they don't prescribe an exact sequence or curriculum.) So planning your instruction sequence carefully can ensure your students continue to build on previous understandings. For example, in Kindergarten, this might mean developing cardinality for numbers 1-5 and writing number names for 1-5 while also concentrating on the count sequence for a limited set of numbers (e.g., 1-10). Later, you can revisit cardinality for an expanded set of numbers (e.g., for numbers 0-10) and writing number names for an expanded set of numbers (e.g., for numbers 0-10) when your students are more fluent with the count sequence for an extended range of numbers (e.g., 0-20). You may iterate again with numbers beyond 10. Addressing skills incrementally requires purposeful sequencing and revisiting standards.
>
> Manipulatives are frequently a large part of the tasks and lessons that follow. **It's important that manipulatives are connected to written methods**, rather than taught as a method for computing or solving unto themselves. Used this way, manipulatives can be an important way to build conceptual understanding of numbers and operations.[3]

Let's take a look at exactly what these standards say.

K.CC.A | Know number names and the count sequence.

K.CC.A.1

Count to 100 by ones and by tens.

K.CC.A.2

Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.A.3

Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

As we've started to discuss, Kindergarten is fundamentally about connecting the ideas of number and quantity (i.e., how many). This begins with knowing number names and the counting sequence, (**K.CC.A**) often before students can count objects or tell how many objects are in a group. This is sometimes referred to as saying the "counting words." Let's take a closer look at counting.

Counting

Having fluency in saying the count sequence allows students to better pair one number name with one object, without being challenged to remember which number is next in the count sequence. Learning to count to 100 builds over the course of the year with daily opportunities to practice (e.g., with a 100 chart or a number line to 100). The task below shows a simple activity that can be done to build fluency with the counting sequence.

Choral Counting

The teacher will need a 100 chart or large number line and a pointer.

As a whole group, have students chant the counting sequence starting with one to thirty, using the pointer to follow the number sequence. Over time, increase the range to one to fifty and then one to one hundred. Eventually have a student take over the job of pointing out the numbers in the sequence. Highlight the multiples of ten using a marker or a colored screen and have students chant the counting sequence by 10s. This should be done daily.

It's important to note the last sentence in the task description: "This should be done daily." Activities that increase facility with knowing number names and the count sequence should happen daily in Kindergarten, as these skills are so fundamental.

> In Kindergarten and first grade, the numbers 11-19 and 20-100 can pose counting challenges for students due to a few irregularities:
>
> - The numbers 11-19 do not consistently make their base-ten meaning evident (e.g., The word "eleven" does not suggest a ten and a one.).
> - For 13-19, the number of ones is said first (e.g., To say "13" the "3" is emphasized first in the word "thirteen.").
> - The number words 20-100 switch to agree the written form (i.e., the number of tens is said first). However, students sometimes have trouble counting across a decade. Instead of counting "29, 30, 31, 32," children make counting errors such as "twenty-nine, twenty-ten, twenty-eleven, twenty-twelve." This is because the decade words (e.g., thirty) do not clearly indicate they mean a number of tens (e.g., thirty is 3 tens) and because the number words "eleven" and "twelve" do not cue students that they mean "1 ten and 1" and "1 ten and 2."
>
> These difficulties underscore the need for daily practice with the count sequence.
>
> Source: Progressions for the Common Core State Standards in Mathematics (draft): Number and Operations in Base Ten, K–5, pp. 5-6.

Counting forward from a given number

A particularly important aspect of knowing the counting sequence is the ability to count forward from a given number (within 100, of course). (**K.CC.A.2**) Counting forward from a given number is a very important prerequisite to "counting on" strategies of addition (**1.OA.C.5**) and, like all new skills, requires practice. Often, students sub-vocalize (i.e., count in their heads, or quietly under their breath) until they reach the given number and then begin counting aloud after the given number; this indicates a need for more practice and flexibility with the count sequence. To build such flexibility, we may structure activities around students naming the "number before" and the "number after." The task below shows one example of this.

Number After Bingo

Each student will need a different Number After Game Board (a 5x5 grid with numbers from 2 through 15 randomly arranged, one in each square), 15 each of two different color counting chips and a set of 2-3 each of number cards with the numbers 1 through 15 on them.


Number AFTER Bingo 1-15

4	10	8	13	8
9	14	2	12	11
15	3	10	5	11
6	13	16	7	9
13	7	11	12	12

Begin whole group by discussing what "number after" means. Next have the students identify and point out on a large number line the number after various numbers selected by the teacher. Initially keep these numbers in the range of 1-15. After the group seems to have an understanding of what "number after" means and how to locate them on the number line, have students play Number After bingo on the 5x5 bingo board in pairs. Students will take turns drawing a number card, stating the number after and placing his/her counter on that number on the game board. The first student with 3 counters in a row on the grid is the winner. As students progress the practice range should be increased by changing the numbers on the grid and the corresponding numbers on the cards.

As the commentary from the task notes, this activity could easily be modified to focus on the "number before."

By the end of Kindergarten, students should be able to count to 100 and write numbers from 0 to 20. They should be able to write numbers in relation to the count sequence as well as write numbers that represent a number of objects. (**K.CC.A.3**)

Number recognition vs. number identification

An important distinction in understanding the written number is that between *number recognition* and *number identification*.

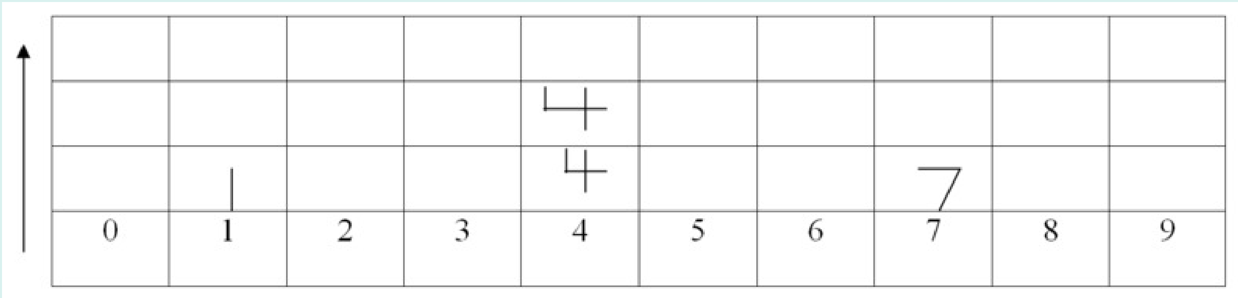
- **Number recognition** is when a student is asked to select a given number from a set of choices. For example, a teacher might ask, "Which is the number 5?" while showing a series of numerals. The teacher says the number name and the student "recognizes" the numeral by pointing to the 5.
- **Number identification** is when the student is asked to come up with the name of a number. For example, a teacher might ask, "What number is this?" while showing the numeral 5. The student responds by saying the number 5. Number identification is a more advanced understanding than number recognition.

How does this distinction play out in classrooms? Students should experience activities in which they practice saying, recognizing, identifying, and writing number names. Some students might be ready for number identification right away, while others may need more practice with number recognition.

The task below supports writing and identifying numbers:

Race to the Top

The teacher will need a sheet of 1 inch graph paper turned sideways (horizontally) with the numbers 0-9 written one in each box in the bottom row along the 11” side of the paper (one copy per student) and a 10 sided number die (0-9) or a 0-9 spinner.



The student rolls a number using the die or spinner and writes that number in the next box of the corresponding column. Students start at the bottom of the page and work to the top. Each time the student rolls/spins a number he/she will write the number on the paper in the next corresponding box. The winning number is the first to make it to the top of the paper. Students can also work in pairs.

You can develop and deepen number identification by asking your students targeted questionsFor example, in the task above, you can ask questions that reinforce number identification, such as:

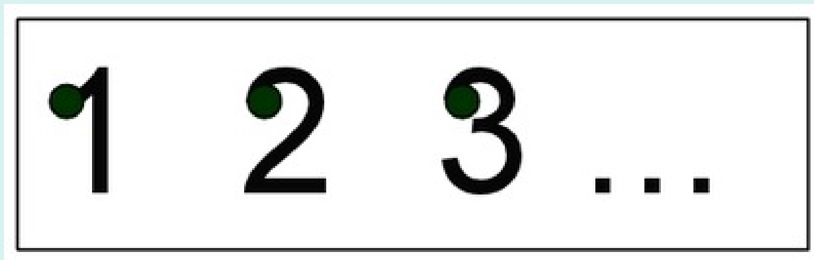
- “Which number is winning?”
- “What number did you roll?”

You can also reinforce counting skills by asking students to order numbers based on the count sequence, as opposed to size. (Kindergarten students do learn that successive number names refer to a quantity that is one larger, but we will discuss that later.) The activity below describes how you can use number lines to support writing and identifying numbers as well as sequencing numbers.

Rainbow Number Lines

The teacher will need a package of regular sentence strips and a medium point black permanent marker. Write the numbers from 1-20 on a sentence strip, one per student. Indicate the starting point for tracing each number with a dot.

As an alternative, the teacher could print out number strips on card stock using the computer. Teacher fonts are available free on-line with traceable number formations.



Give each student five or six different colored crayons (any color but black). Starting at the dot, have them trace over each number with each of the colored crayons. Laminate each number strip and tape it to the student's desk or table area for future reference. Students can also cut the number strip into separate cards to use for sequencing activities.

As described in the commentary for this task, studentscan cut the number line into number cards and practice sequencing the number cards. Alternatively, they can practice sequencing small sets of number cards (e.g., 1-10 or 1-20) while using the number line for support. Students should continue with sequencing activities until they can sequence numbers without support.

The discussion and activities above highlight learning number names and the count sequence. This learning is essential for connecting number names to quantity. The second cluster in the Counting & Cardinality domain is about making this connection. Let's read the standards associated with this cluster, and then we'll think through what they mean and how they look in practice.

K.CC.B | Count to tell the number of objects.

K.CC.B.4

Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.B.5

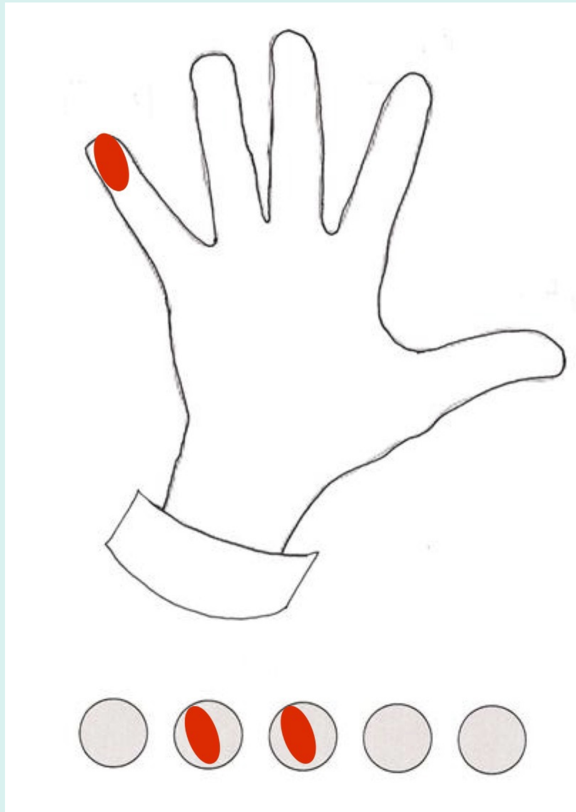
Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

So what does it mean to teach cardinality? Let's take a closer look.

Pairing number names with numbers of objects

Connecting numbers with quantities is a fascinating process, one helped along with concrete materials and activities. Students can point to objects or move them as they say the count sequence, carefully pairing each number name said with one and only one object. (**K.CC.B.4.A**)In the example below, students count beans to practice pairing number names to objects.

Kindergarten, Module 1, Lesson 1: Fluency Practice



T: Take 1 bean out of your bag, and put it on your mat. Count how many beans are on your mat.
S: 1.
T: Take another bean out of your bag, and put it on your mat. Count how many beans are on your mat now.
S: 1, 2.
T: Yes. Take another bean out of your bag, and put it on your mat. Count how many beans are on your mat now.
S: 1, 2, 3.

T: Yes. Let's touch and count them one at a time like this: 1, 2, 3.

S: 1, 2, 3 (touch each bean).

T: Move 1 bean to the pinky fingernail. How many fingers have a bean?

S: 1.

T: How many fingernails are under the bean?

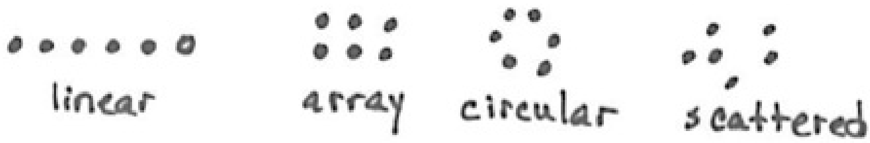
S: 1.

T: Is that exactly the same number?

S: Yes!

This task may appear very simple, but something very important is happening! Students are pairing number names, "1, 2, 3" with corresponding quantities of beans. Because this act of pairing is foundational for developing cardinality you may want to begin early in the year, starting with sets of objects less than or equal to 5.

As we know, it's easiest to count objects arranged in a straight line. With more practice and facility with one-to-one correspondence (i.e., pairing one number name with one and only one object) students learn to count less structured arrangements of objects, such as rectangular arrays (making sure not to skip or repeat rows); circles (making sure to stop just before the object they started with; and, lastly, scattered configurations (making sure not to skip or double count objects).[4]



Source: EngageNY Kindergarten, Module 1, Overview

As we've discussed, fluency with number names and the counting sequence(**K.CC.A**)is important for developing an understanding of cardinality. It allows students, when they're counting objects, to focus on pairing one number name to one object rather than trying to remember which number comes next in the count sequence.[5] For example, students who don't know the count sequence through 5, may be challenged to pair each object with a successive number name. As they increase their fluency with the count sequence, they can increase the number of objects they can count.

Counting to tell "how many"

As students become more practiced and proficient in counting objects, they come to understand that the last number name they say in their counting tells them the number of objects they've counted (i.e., how many?). (**K.CC.B.4b**) While it may seem odd to see this spelled out in the standards, it's an important understanding that may not be readily clear to students (i.e., "If I start counting objects and end at the number 7, there are 7 objects). As described in the Progression, "Prior to reaching this understanding, a student who is asked 'How many kittens?' may regard the counting performance itself as the answer, instead of answering with the cardinality of the set."[6] So if there are 4 kittens, a student might consider the answer to "how many?" to be the performance of counting 1, 2, 3, 4, rather than the cardinality of the set, which is 4. Students may need support in making the explicit connection that the last number said represents the number of objects. Questions like, "How many kittens did you count?" followed by, "So, how many kittens are there?" might help make this connection.

In addition, students who have a lot of experience and discourse around counting different arrangements of objects will more readily understand that the number of objects is the same regardless of their arrangement or the order in which they were counted (**K.CC.B.4b**) In the lesson below, students count cubes to find how many and then arrange the cubes in different configurations and verify that the count is the same.

Kindergarten, Module 1, Lesson 7: Concept Development

T: Good counting and finding, everyone! (Hold up a bag of non-connected cubes.) Look at these cubes! I want to count how many I have of each color cube, but they are all mixed up! What should I do?

S: Let's dump them out and put the cubes that are the same color together.

T: Good idea! (As a whole group, work together to connect the same colored cubes. Position the sticks of connected cubes vertically.)

T: Now we can count how many of each color cube we have. Let's count the blue cubes.

S: There's only 1 blue cube.

T: Yes. What card can we put under the blue cube to show that there is only one blue cube?

S: The number 1. (Call on a student to choose the correct card and place it beneath the blue cube.)

Continue until all of the numeral cards 1–5 are placed under a stick of cubes.

T: (Turn the 5-cube stick horizontal.) Do we have to change the numeral card for this stick of cubes? (Provide wait time, and call on several students. Be sure to ask the reason why or why not.)

S: No, because there are still 5 cubes, so the 5 card is still good.

T: Should we count the cubes again? (Provide wait time, and elicit several opinions. Be sure to ask the reason why or why not.)

S: Yes, we should, just to make sure. / No, we don't have to because you didn't put any more cubes on or take any off; you just turned the stick.

T: Okay. You are ready to try this at your desk. (Distribute materials. Monitor how each student organizes her cubes and digit cards, horizontally, vertically, or both horizontally and vertically.)

Kindergarten, Module 1, Lesson 7 Available from engageny.org/resource/kindergarten-mathematics-module-1-topic-c-lesson-7; accessed 2015-05-29.

Students should be able to count as many as 20 objects in various arrangements (10 objects in a scattered arrangement), and also, given a number 1-20, they should be able to count out that many objects.(**K.CC.B.5**) Counting out objects is a more advanced skill because students must have enough facility with counting that they are able to sustain attention on the number of objects to be counted out.[7]

In Kindergarten, students should have lots of experiences counting to answer "how many?" questions. As in the task below, activities should incorporate opportunities for students to write the numeral that represents the number of objects. (**K.CC.A.3**)

Goody Bags

Materials

- Many small ziplock bags of counting objects (the "goodies"). Each bag should contain a number of objects in the counting sequence students are working on, between 1 and 20.
- Post-it notes and pencils.

Actions

Students count the objects, record the number on the post-it note and stick the post-it note onto the outside of the bag.

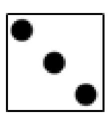
Students can work in pairs to scaffold their counting, and teachers should take care that at least one student in each pair is a confident counter and at least one student is a confident writer. Students can be provided with a number line to aid in writing the numbers. If the teacher is concerned that students cannot write the numbers independently s/he can write the quantities on post-it notes ahead of time and let the students choose the post it note that goes with each goody bag.

It's important to point out that students should have experience thinking about the size of objects in relation to the number of objects. As noted in the commentary section for the "Goody Bags" task above, some bags should contain a small number of large objects and some should contain a large number of small objects. Experiences like this help students understand that small objects don't "count less" than large objects.

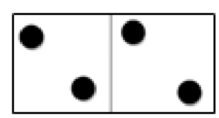
Subitizing

Quick: how many dots are there in each picture below?

Picture 1



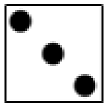
Picture 2



Did you have to count? Or did you just "know"?

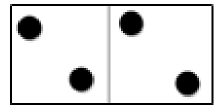
With lots of practice counting small groups of objects, with varying arrangements, students begin to recognize the cardinalities of small groups without having to count each object in the group. For example, a student might roll a three using a six-sided cube and know that three dots is the number 3, without needing to count the three dots individually, as in Picture 1 above. This is referred to as **perceptual subitizing**. Perceptual subitizing is important because it leads to **conceptual subitizing**, the ability to recognize that a collection of objects is composed of subcollections, and that by quickly combining the cardinalities of the subcollections, you can find the cardinality of the collection. In other words, a number is composed of other numbers. For example, a student might see a domino with two subsets, each with a cardinality of 2, and know the cardinality for the whole set is 4.[8]

perceptual subitizing



knowing there are 3 dots without counting each dot

conceptual subitizing



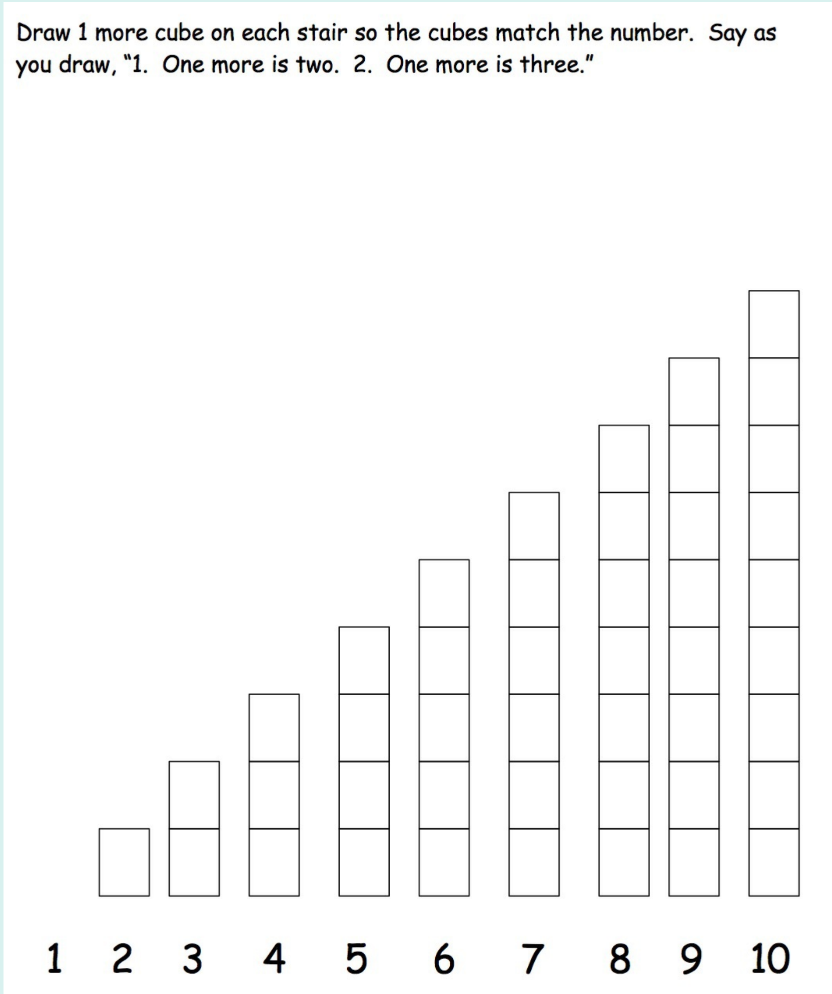
knowing the cardinality is 4 based on noticing two subsets of 2

Subitizing is important because it supports addition and subtraction strategies, including“counting on,” and eventually leads to more sophisticated methods for supporting all four operations. We’ll explore conceptual subitizing more when we discuss decomposing numbers in the Operations & Algebraic Thinking cluster **K.OA.A**.

Laying a foundation for “counting on”

The foundation for “counting on”—an addition and subtraction strategy more sophisticated than counting all—begins in Kindergarten, and there are several concepts related to counting and cardinalitythat support “counting on” strategies. Students have to have facility with counting forward from a given number (**K.CC.A.2**) as well as with determining the cardinality of a group of objects. Equally important is understanding that each successive number name refers to a quantity that is one larger. (**K.CC.B.4c**) Students combine these understandings in order to understand “one more” and “one less” without having to recount the number of objects in the group to which an object was added or subtracted. Relating each successive number name to “one more” or “one less” in a concrete way can help students grasp the concept. For example, in the lesson below, your students can work with linking cubes to discover that each number tower is one more than the previous number tower. The activity asks them to draw “one more” cube on each tower so that the number of cubes in the tower is equal to the number beneath the tower. Students also use the language of “one more” to relate successive numbers.

Kindergarten, Module 1, Lesson 30: Homework



Kindergarten, Module 1, Lesson 30 Available from engageny.org/resource/kindergarten-mathematics-module-1-topic-g-lesson-30; accessed 2015-05-29.

The strategy of "counting on" builds from these understandings in Kindergarten. Eventually, in first grade, a counting word represents a group of objects that are added or subtracted and the addends are embedded in the total.[9]

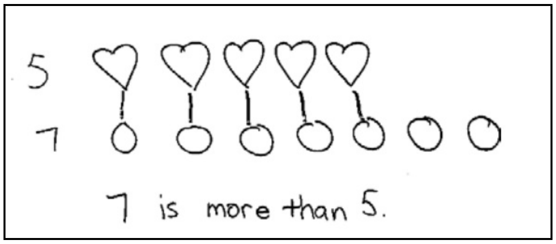
The third cluster in the Counting & Cardinality domain deals with comparing numbers. Let's read the standards associated with this cluster, and then we'll think through what they mean and how they look in practice.

K.CC.C | Compare numbers.

> **K.CC.C.6**
> Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
>
> **K.CC.C.7**
> Compare two numbers between 1 and 10 presented as written numerals.

Comparing numbers

Comparing numbers is closely connected to counting and cardinalityWhen students begin doing this, they should use actual objects or manipulatives and make comparisons between groups of objects. (**K.CC.C.6**) However, by the end of the year, they should be able to compare numbers given in only the written form. (**K.CC.C.7**) Students begin comparing numbers of objects in two groups using strategies such as matching. Matching strategies allow students to pair objects and see if there are any extra (i.e., more) and to concretely determine "which is more" and "which is less". The example below illustrates matching as a strategy to determine which number is more (or less).



Source: [EngageNY Kindergarten, Module 3, Lesson 25]

The dialog in this lesson uses questioning to develop the matching strategy for comparing numbers.Also, note the emphasis on using concrete objects; students compare, say, five objects and seven objects, rather than the numerals alone. Matching is an important strategy that lends nicely to compare problems in Grade 1 (i.e., "How many more?" "How many less?")(**1.OA.A.1**) and understanding even and odd numbers in Grade 2. (**2.OA.C.3**)

> It's important that students practice comparing different sized objects. Students need to learn that even if one group looks like it has more (or bigger) objects, that matching and/or counting might yield a different result.[10]

The abstract concept of comparing numerals

After students show facility with comparing groups of objects, you can begin to encourage them to compare numbers presented as numerals. This is an abstract concept for students, so it's a good idea to give them alot of practice with concrete and pictorial comparisons beforehand. The task below shows how students might compare numbers presented as numeralsonly.

Guess the Marbles in the Bag

This activity can be done as an entire class or in small groups.

Materials
Paper bags and marbles (or some other counter, as long as it is relatively noisy).

Actions
(Whole-class version) The teacher secretly places between 1 and 10 marbles in a paper bag, then shows the bag to the class. After shaking it enough times for students to hear the marbles inside, and 4 or 5 students guess how many marbles are in the bag. The teacher writes the guesses on the board. Afterwards the contents of the bag are revealed and counted out. The teacher writes the number representing the total on the board, and the students then help sort their guesses into less than, greater than, or equal to the number of marbles in the bag. The game repeats until everyone has had a chance to guess at least once.

(Small group version) This works like the class version but one student in a group fill the bag with marbles themselves and the rest of the group tries to guess the number. With this variation it is practical to allow the students to both hear and feel the marbles inside the bag before they make their guess.

It's important for students to articulate their reasoning for making comparisons so that we may understand how students are thinking. The sample solution below shows the types of responses students should be giving to demonstrate their understanding when comparing numbers. Also note that the teacher asks the students to turn and talk to a partner and explain how they know which number is greater or less than the other number.

Which number is greater? Which number is less? How do you know?

Solution

"When I count it goes 1, 2, 3, 4, 5, *6*, 7, 8, then *9*. I know that six is less than nine because I say six before I say nine."

"I know that nine is greater than six because my brother is nine years old and I'm six years old, and he's older than me."

"Okay, you have six. One more is seven. One more is eight. One more is nine. So nine is greater than six. Okay?"

"Well, when I see the number line I see six comes before nine."

"This many fingers is six (shows six fingers). This many fingers is nine (shows nine fingers). I have to put up more to show you nine."

Part 2: How does Counting & Cardinality relate to other parts of Kindergarten?

There are lots of connections among standards in Kindergarten; if you think about the standards long enough, you'll probably start to see these relationships everywhere. In this section, we'll talk about the connection between the Counting & Cardinality standards and the standard in the second cluster of the Measurement & Data (MD) domain. This MD cluster is a supporting cluster and can be used to support work with the CC standards.

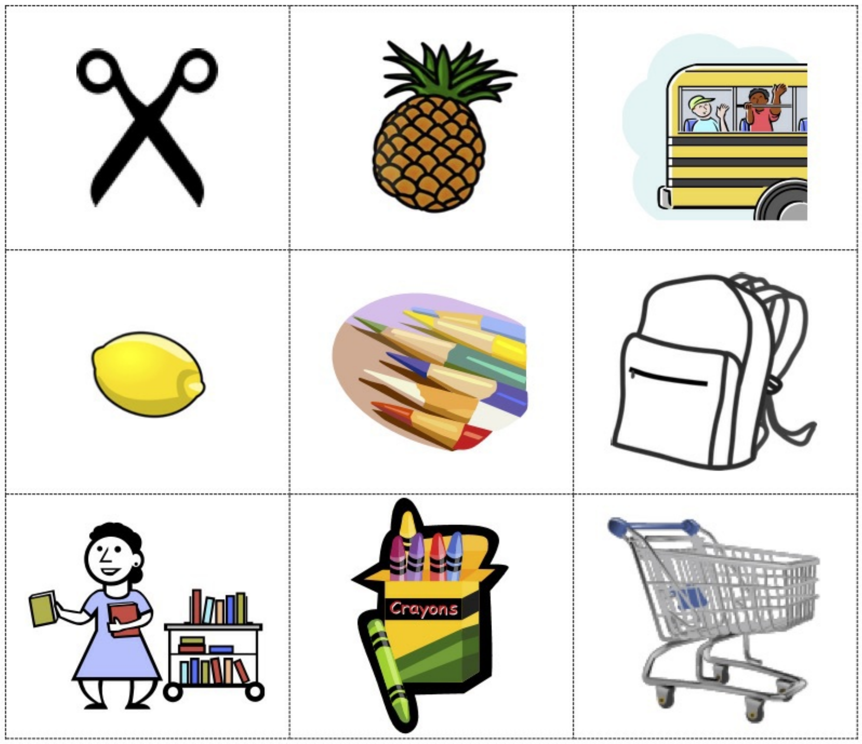
☐ K.MD.B | Classify objects and count the number of objects in categories.

> **☐ K.MD.B.3**
> Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.*
>
> **Limit category counts to be less than or equal to 10.*

This standard can be thought of as a context for reinforcing the major ideas of Counting & Cardinality. In fact, even the language of the standard, "count the number of objects in each category," signals the intent to classify objects for the purposes of counting groups of objects. This might be a departure from previous teaching of classification, which may have been treated as a separate set of skills; the emphasis here is on using classification to support counting. For example, in the activity below, students cut and glue pictures onto a library, school, or grocery store, then write how many objects are associated with each location.

Kindergarten, Module 1, Lesson 5: Homework

Cut and glue where each belongs. Write how many.

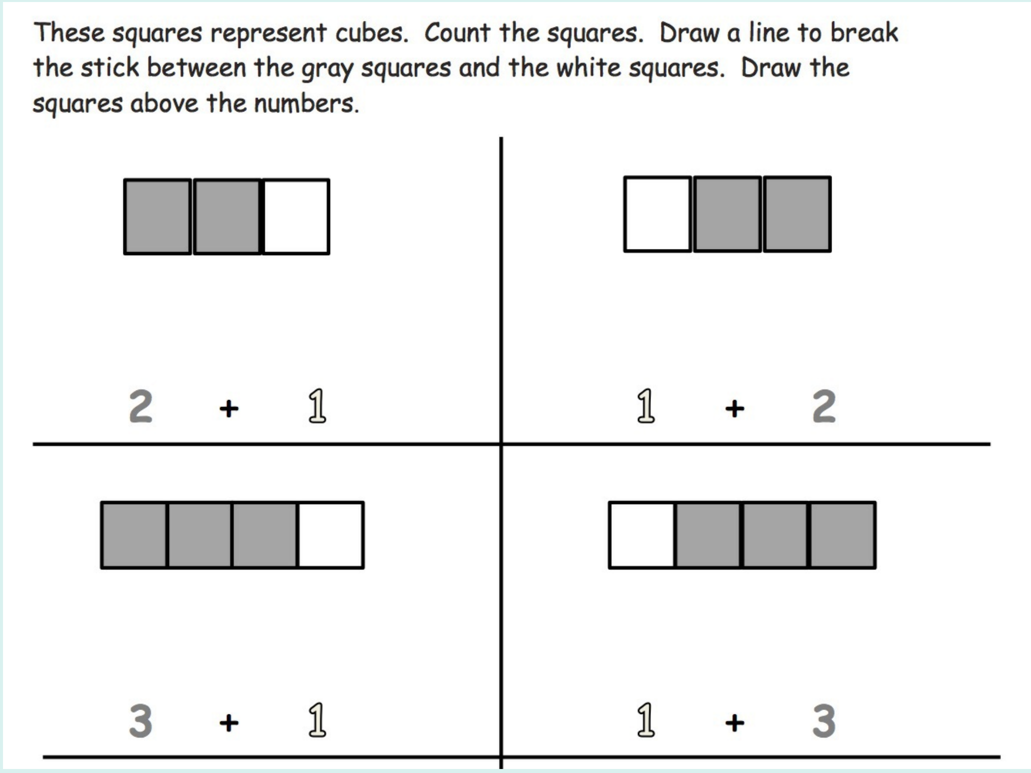


Kindergarten, Module 1, Lesson 5 Available from engageny.org/resource/kindergarten-mathematics-module-1-topic-b-lesson-5; accessed 2015-05-29.

Also, students can compare the number of objects in the classified groups. In the above example, we could ask "Are there more objects at the library than the school? How do you know?" to reinforce understandings of **K.CC.C**.

Beyond the logical connection to **K.MD.B**, K.CC work will connect meaningfully to the skills and understandings in the Operations & Algebraic Thinking domain (OA). In particular, students will rely on counting as they begin work decomposing numbers within 10. (**K.OA.A.3**) In the example below, students count the total, then decompose the total and relate the decomposition to an addition expression.

Kindergarten, Module 1, Lesson 11: Problem Set



Helping students to see that a total is composed of smaller cardinalities (e.g., 4 squares are composed of 3 gray squares and 1 white square) supports students' understanding of addition. (**K.OA.A.3**)

Finally, K.CC will lay the foundation for work with Number & Operations in Base Ten(NBT). Students will use counting as they explore composing and decomposing numbers from 11 to 19 into ten ones and some further ones. (**K.NBT.A.1**) Students will use counting up from 10 (**K.CC.A.2**)and understanding that the last number named is the number of objects(**K.CC.B.4c**)when decomposing teen numbers. In the task below, students use counting to illustrate the decomposition of teen numbers into ten ones and some more ones.

What Makes a Teen Number

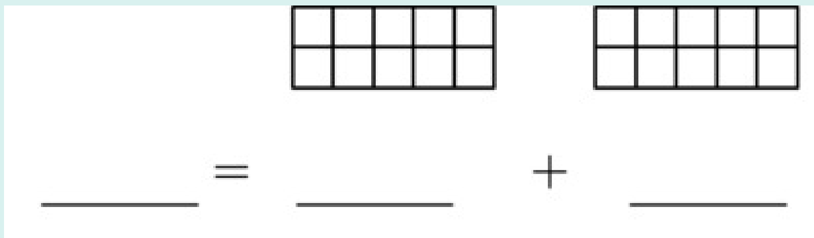
Decompose teen numbers using 10-frames and a number equation.

Materials

- Number cards 11-19
- Pencil, crayon, or marker
- Attached student worksheet

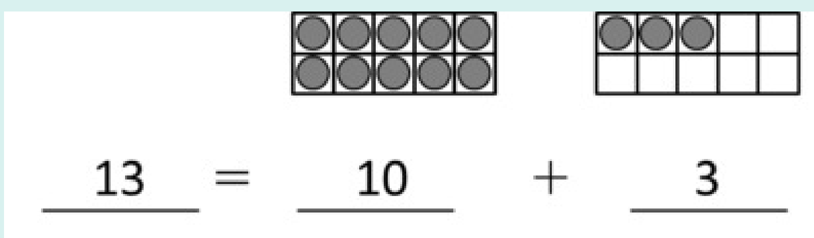
Action

This activity can be done individually, in partners, or in small groups. The students have a teacher-made sheet and a writing implement. The cards are shuffled and placed face down.



The student picks a card off of the top of the pile. The student then says the number and draws that many dots beginning with the first 10-frame. When the first 10-frame is filled, the student continues drawing the remaining dots in the next 10-frame. The student then fills in the blank equation with the corresponding numbers.

Example:



The student continues to pick cards and illustrate numbers in this way until all cards are used or the sheet is filled.

To reinforce the connection to K.CC, we might ask students to:

- Count out loud and point to the dots while counting.
- Explain how the count tells them how many dots there are

Part 3: Where is Counting & Cardinality going?

Let's take a look at where the foundations of Counting & Cardinality and classifying objects in Kindergarten will take us.

Podcast clip: Importance of Coherence with Andrew Chen and Peter Coe (start 9:34, end 26:19)

Foundations for Operations & Algebraic Thinking and Number & Operations in Base Ten

As we've mentioned, the Counting & Cardinality standards in Kindergarten provide a significant conceptual basis for almost all later work within the Operations & Algebraic Thinking (OA) and Number & Operations in Base Ten (NBT) domains. Together, these two domains comprise the heart of the mathematics students learn in Kindergarten through Grade 2.

From K.CC to Operations & Algebraic Thinking in Grades 1 and 2

The most essential basis for further work is rooted in being able to count forward from a given number and understanding that each successive number name refers to a quantity that is one larger. (**K.CC.A.2**,) (**K.CC.4c**) These abilities will be vital for students to be able to use "counting on" strategies for addition and subtraction in Grade 1. (**1.OA.C.5**) For example, in the lesson below, students "count on" from 4 to determine the total number of apples, 6. Instead of counting all 6 apples beginning with 1, students count on 2 more from 4 to get 6.

Grade 1, Module 1, Lesson 4: Problem Set

T: Let’s look at the picture of 6 apples and use our beans to find different ways to make 6.
T: How many apples do you see?
S: 6.
T: Let’s see how many apples with stems are there. Put a red bean on each apple as we count.
S: 1, 2, 3, 4.
T: How many apples do not have stems? Let’s put a white bean on each stem-less apple and count.
S: 1, 2.
T: Let’s see how many apples there are by counting on from the red beans. As you count, touch each bean.
S: Foooour, 5, 6.

Grade 1, Module 1, Lesson 4 Available from engageny.org/resource/grade-1-mathematics-module-1-topic-b-lesson-4; accessed 2015-05-29.

Counting on, as well as other strategies, leads to fluency with addition and subtraction within 10 in Grade 1,[1.OA.B.6] and fluency within 20 in Grade 2.[2.OA.A.2]

From K.CC to Number & Operations in Base Ten in Grades 1 and 2

In Grade 1, students will extend the counting sequence to 120 and again, be able to relate these numbers to “how many” questions. (**1.NBT.A.1**) From here, students will come to understand the concept of a bundle of ten ones called a “ten” and how “ten” is expressed in numbers like 14 and 40. (**1.NBT.B.2**) In this task, students use manipulatives to represent tens and ones:

Roll & Build

Materials

For each pair:

- 2 ten-sided dice with the numbers 0 to 9 or two spinners with the numbers 0 to 9
- Base-10 blocks, linking cubes, or bundled and loose popsicle sticks
- Paper and pencil

Play

- Student A rolls the dice.
- Student B makes a number using the values on the dice as digits and both students write it on the paper. For example, if student A rolled a 3 and a 4, the number can be 34 or 43.
- Student A represents the number with the tens and ones blocks/popsicle sticks.
- Student B counts the blocks to check that they correctly represent the number.
- Both students draw a picture of the tens and ones on the paper.

The students should take turns.

Success in a task like this depends on fluency with the count sequence, as well as cardinality. Students must be able to fluently identify numbers and count out manipulatives. Note also the way that work with manipulatives is connected to written work (i.e., students write down a number and then represent the number with blocks or popsicle sticks). This understanding is expanded again when students learn about hundreds in Grade 2. (**2.NBT.A.1**)

Foundations for further work in Measurement & Data

Work with data will continue, based on the foundation of students classifying objects and counting the number of objects in each category. (**K.MD.B.3**) Students continue this in Grade 1 by organizing data into three categories and answering comparison questions. (**1.MD.C.4**) Students go further in Grade 2 by solving problems about bar and picture graphs(**2.MD.D.10**) The work with data organization in Grades K-2 is meant to support the major work in the Counting & Cardinality and Operations & Algebraic Thinking domains.

The Illustrative Math and EngageNY Tasks below illustrate thisprogression:

Kindergarten

Sort and Count 1

You will need sorting cards or items, for example: colors, shapes, animals, foods, etc. Cards should be able to be sorted multiple ways (example, foods could be sorted by color, then sorted by fruit vs. veggie vs. grain). Another example is animals could first be sorted by pet vs. wild animal vs. farm animal and next be sorted by number of legs and finally be sorted by furry animals/skin animals/scale animals.

First have students look at the cards and decide two or three different ways to sort. Next each student can randomly choose a card or item. Then when all class has one, they sort themselves into categories according to color, shape, type of animal or food they have. Then the teacher can ask the questions:

- "Which group has the most?"
- "Which group has the least?"
- "Do any groups have the same number?"

The students count the groups and answer the teacher's questions.

Grade 1, Module 3, Lesson 10: Exit Ticket

A group of students were asked what they ate for lunch. Use the data below to answer the following questions.

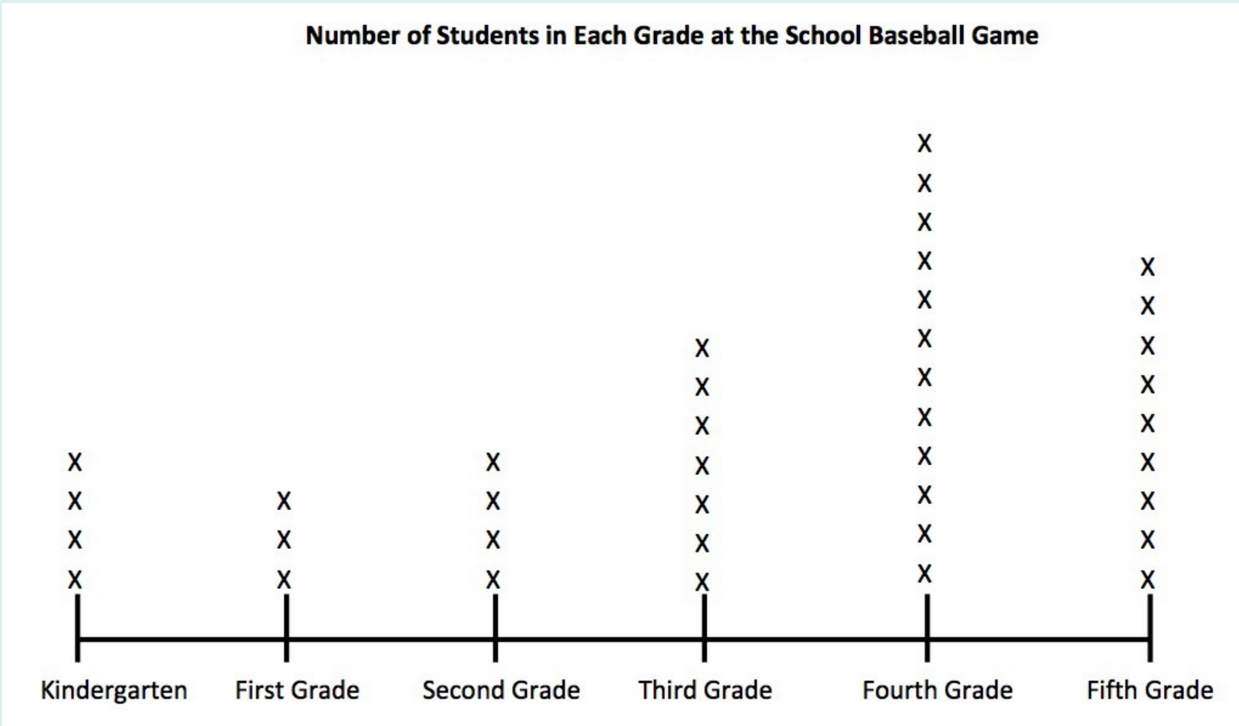
Student Lunches

Lunch	Number of Students
sandwich	3
salad	5
pizza	4

1. What is the **total** number of students who ate pizza?
2. Which lunch was eaten by the **greatest** number of students?
3. What is the total number of students who ate pizza or a sandwich?
4. Write an addition sentence for the **total** number of students who were asked what they ate for lunch.

Grade 2, Module 7, Lesson 25: Exit Ticket

Answer the questions using the line plot below.



1. How many students went to the baseball game?
2. What is the difference between the number of first-grade students and the number of fourth-grade students who went to the baseball game?
3. Come up with a possible explanation for why most of the students who attended are in the upper grades.

Congratulations on completing our Kindergarten guide for Counting & Cardinality and Classification of Objects. We hope it provided a clear and detailed explanation of what these standards say, and how Counting and Cardinality relates to other mathematical concepts in Kindergarten, as well as Grades 1 and 2.

The content here can be helpful when writing or evaluating a scope and sequence, a unit plan or lesson plan. And you can find additional resources here:

Student Achievement Partners: Focus in Kindergarten

Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking

EngageNY: Kindergarten Module 1 Materials

Illustrative Mathematics Kindergarten Tasks

Endnotes

[1] The Common Core State Standards for Mathematics (CCSSM) are organized into major, additional, and supporting clusters in the Focus by Grade Level documents from Student Achievement Partners.

[2] Standards for Mathematical Practice: Commentary and Elaborations for K–5, p. 9

[3] See the K-8 Publishers' Criteria for the Common Core State Standards for Mathematics, p. 18

[4] See the Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 4.

[5] See the Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 4.

[6] See the Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 4.

[7] See the Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 4.

[8] Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 4.

[9] Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 5.

[10] Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 5.